Particle Accelerators

Eric Prebys, FNAL
Summer Intern Talk
Fermilab
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Acknowledgments

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  - Susan Winchester, Elvin Harms, and Sam Posen
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- Google and Wikipedia, for making us all scholars
- Fermilab, for encouraging outreach
- You, for coming
A little about me...

- **Home town:** Phoenix, Arizona
- **1984:** BS in Engineering Physics, University of Arizona
  - Got a job in an HEP group (after being fired from a gas station).
- **1984-1990:** Grad Student, University of Rochester
  - PhD topic: Photon Production in Hadronic Interactions (FNAL)
- **1990-1992:** CERN Fellow, CERN (Geneva, Switzerland)
  - Worked on OPAL Experiment at LEP e⁺e⁻ collider
- **1992-2001:** RA and Assistant Professor, Princeton Univ.
  - Worked on Superconducting Super Collider (Texas)
  - Belle e⁺e⁻ Experiment at KEK, Japan
  - Laser-electron scattering experiment at SLAC (Stanford)
- **2001-Present:** Scientist, Fermilab
  - Past:
    - MiniBooNE short baseline neutrino oscillation experiment
    - Proton Source (Linac+Booster) Department Head
    - Director of LHC Accelerator Research Program (LARP)
    - Director of Joint University-Laboratory PhD Program
  - Present:
    - Mu2e rare muon conversion experiment
    - Integrable Optics Test Accelerator (IOTA) proton injection
    - Program director for Lee Teng Undergraduate Internship
    - Occasional Instructor at US Particle Accelerator School
- **Sep. 2017-?:** Professor of Physics, UC Davis
  - Mu2e
  - Director of Crocker Nuclear Laboratory

Experimental HEP
Accelerator Physics (mostly)
Bit of both
We will talk mostly about the quest to reach the highest energy in accelerators, because that has driven the technology.

In fact, high energy accelerators make up only a tiny fraction of the accelerators in the world.

We’ll summarize some of the many other applications of accelerators near the end.

Like all speakers, my talk represents my experience and expertise:

- Another speaker might emphasize different things.
Energy is \((\text{force}) \times (\text{distance})\)

For example, when you drop something, gravity “work” through the change in height to convert “potential energy” to “kinetic energy”.

\[
\Delta \text{PE}_{\text{elec}} = \Delta \text{KE}
\]

\[
\Delta \text{PE}_{\text{grav}} = \Delta \text{KE}
\]

\[
(\text{kinetic energy}) = (\text{charge}) \times (\text{voltage})
\]

In the same way, when we accelerate something in an electric field, electrical potential (“voltage”) is converted to kinetic energy.

For this reason, a convenient unit of energy is the “electron-volt (eV)” which is the energy you get when you accelerate a charge of one electron (or proton) over a 1 Volt potential.
The eV is a *really small* unit of energy.
- $1.6 \times 10^{-19}$ (=.0000000000000000016) Joules - our usual unit of energy.
- A 1 kg weight dropped 1m would have $6 \times 10^{19}$ $(6000000000000000000)$ eV of energy!

On the other hand, it’s a very useful unit when talking about individual particles
- If we accelerate a proton using an electrical potential, we know exactly what the energy is.
- It’s also useful when thinking about mass/energy equivalence
  - Which leads us to...
Momentum and energy in special relativity

Classically:

momentum: \( \vec{p} = m\vec{v} \)

force: \( \vec{F} = m\vec{a} = m\frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t} \)

kinetic energy: \( K = \frac{1}{2}mv^2 = \frac{p^2}{2m} \)

Relativistically:

Rest energy: \( E = mc^2 \)

momentum: \( \vec{p} = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} m\vec{v} \equiv \gamma m\vec{v} \)

force: \( \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = m\frac{\Delta (\gamma \vec{v})}{\Delta t} \) (both \( v \) and \( \gamma \) change)

total energy: \( E^2 = (mc^2)^2 + (pc)^2 = \gamma mc^2 \)

kinetic energy: \( K = E - mc^2 = (\gamma - 1)mc^2 \)

Always right if you write it this way

This is new!

Visualizing the relationship
Mass and Energy

- High Energy Physics is based on Einstein’s equivalence of Mass and Energy
  \[ E = mc^2 \]
- All reactions involve some mass changing either to or from energy

Chemical Explosion

.00000005% of mass converted to energy.

Hydrogen Bomb (fusion)

~.1% (of just the Hydrogen!) converted.

- If we could convert a kilogram of mass entirely to energy, it would supply all the electricity in the United States for almost a day.
The electron and proton have very tiny masses in our usual units

\[ m_e = 9.11 \times 10^{-31} \text{ kg} \]

\[ m_p = 1.67 \times 10^{-27} \text{ kg} \approx 2000 \times m_e \]

However, they are more reasonable if we express them in terms of their rest energy in eV:

\[(\text{proton mass}) \times c^2 = 938,000,000 \text{ eV} \approx 1 \text{ billion eV} = 1 \text{ GeV}\]

\[(\text{electron mass}) \times c^2 = 511,000 \text{ eV} \approx \frac{1}{2} \text{ MeV}\]

These energies establish a natural scale to which we can compare beam energy.
A body in motion will have a total energy given by

\[ E = \sqrt{mc^2 - \left(\frac{v}{c}\right)^2} \equiv \gamma mc^2 \]

The difference between this and \( mc^2 \) is called the “kinetic energy”

Here are some examples of kinetic energy

<table>
<thead>
<tr>
<th>Example</th>
<th>Velocity</th>
<th>Velocity/Speed of light</th>
<th>Kinetic Energy/(mc^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Race car</td>
<td>150 mph</td>
<td>.00000002</td>
<td>.000000000000000025</td>
</tr>
<tr>
<td>Apollo 12 (fastest people)</td>
<td>24,791 mph</td>
<td>.0000037</td>
<td>.00000000000068</td>
</tr>
<tr>
<td>Fermilab LINAC (K=400 MeV)</td>
<td>214,000,000 m/s</td>
<td>.71</td>
<td>.43</td>
</tr>
<tr>
<td>Proton in the LHC (full energy)</td>
<td>Light minus 2.7 m/s</td>
<td>.9999999991</td>
<td>7500</td>
</tr>
<tr>
<td>Electron in LEP</td>
<td>Light minus 3.6 mm/s</td>
<td>.99999999988</td>
<td>203,000</td>
</tr>
</tbody>
</table>

\( c = \text{(speed of light)} = 300,000 \text{ km/s}! \)

For \( v \ll c \) (speed of light), Kinetic energy \( \sim \frac{1}{2}mv^2 \)

\[ \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \]
Question: Why are “blue ray” players blue?

Answer: because light is “quantized”* and blue light is more energetic and has a shorter wavelengths, so the “bits” can be smaller

\[ \lambda = \frac{hc}{E} \]

*See SMP talk by Paddy Fox, Jan. 21, 2017
It turns out that all particles have a wavelength

\[
\lambda = \frac{h}{p} \approx \frac{(\text{size of a proton})}{\text{Energy (in GeV)}}
\]

So going to higher energy allows us to probe smaller and smaller scales

If we put the high equivalent mass and the small scales together, we have...
Going to higher energies = going back in time
Accelerators allow us to go back 13.8 billion years and recreate conditions that existed a few trillionths of a second after the Big Bang. The place where our current understanding of physics breaks down.

In addition to high energy, we need high “luminosity” that is, lots of particles interacting, to see rare processes.
Built at CERN, straddling the French/Swiss border
27 km in circumference
Currently colliding proton beams at 6500 GeV ($6.5 \times 10^{12}$ eV) each
That’s where we are. Now let’s see how we got here...
The main parts of an accelerator

- A source of particles
  - Electrons
    - Filament
    - Laser->surface
    - Radiofrequency (RF) “gun”
  - Protons/ions
    - Plasma (gas heated until electrons and nuclei separate)

- A method of acceleration
  - Electric fields
    - Static fields
    - Radio Waves (RF)
    - Lasers
    - “Wakefields” in matter
  - Radio Waves (RF)
  - Lasers

- A way to store and focus beams
  - Magnetic fields
    - Cyclotrons
    - Synchrotrons
    - Magnetic focusing
Rewind: some pre-history

- The first artificial acceleration of particles was done using “Crookes tubes”, in the latter half of the 19th century
  - These were used to produce the first X-rays (1875)
  - At the time no one understood what was going on
- The first “particle physics experiment” told Ernest Rutherford the structure of the atom (1911)

- In this case, the “accelerator” was a naturally decaying $^{235}\text{U}$ nucleus

Study the way radioactive particles “scatter” off of atoms

E. Prebys: Particle Accelerators
Radioactive sources produce maximum energies of a few million electron volts (MeV).

Cosmic rays reach energies of ~1,000,000,000 \times \text{LHC} but the rates are too low to be useful as a study tool.

- Remember what I said about “luminosity”.
The simplest accelerators accelerate charged particles through a static electric field. Example: vacuum tubes (or CRT TV’s)

Limited by magnitude of electric field:
- TV Picture tube ~keV
- X-ray tube ~10’s of keV
- Van de Graaf ~MeV’s

Solutions:
- Alternate fields to keep particles in accelerating fields -> Radio Frequency (RF) acceleration
- Use magnetic fields to bend particles so they see the same accelerating field over and over -> cyclotrons, synchrotrons
Magnetic fields

- Magnetic are produced by electric currents, according to the “right hand rule”

Wire:
- put thumb along direction of current
- Field circles wire in direction of fingers

Coil ("solenoid"):
- Wrap fingers in direction of current.
- Field points in direction of thumb.

- Units are “Tesla” [T]
  - 1 T pretty big for normal magnets
  - LHC superconducting magnets go to ~8 T
Moving charged particles in a magnetic field experience a force which is

- Proportional to the charge, magnetic field, and velocity
- is perpendicular to both the field and the direction of motion, with a direction given by the “right hand rule”

Note: force reverses for negative charge!
Because the force is always perpendicular to the direction of motion, magnetic fields can only change the direction of a particle. The velocity (and therefore the kinetic energy) remain constant!

When the direction of motion changes, the direction of the force changes to remain perpendicular.

- circular motion

\[ \vec{F} = q\vec{v} \times \vec{B} = m\vec{a} \rightarrow qvB = m \frac{v^2}{\rho} \]

Note: in 3D, motion is “helical”
The tube generates an electron beam using a hot filament/cathode, “Wehnelt Cylinder”, and accelerating anode.

\[ eV = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2eV}{m}} \propto \sqrt{V} \]

\[ V = 150V \rightarrow v = 7.3 \times 10^6 \text{ m/s} = 0.024c \]

Filled with low pressure Hydrogen, which fluoresces when electrons pass through

Heated filament (electron source)

Wehnelt Cylinder focuses electrons

\[ V = 100 \text{ to } 300V \]

\[ -10 \text{ to } -50V \]
The Helmholtz Coils produce a ~uniform magnetic field

\[ B = \left( \frac{4}{5} \right)^{\frac{3}{2}} \frac{\mu_0 NI}{R} \propto I \]

\[ \rho = \frac{mv}{eB} \]

\[ \propto \frac{v}{B} \]

\[ \propto \frac{\sqrt{V}}{I} \]
The Cyclotron (1930’s)

- A charged particle in a uniform magnetic field will follow a circular path of radius

$$\rho = \frac{p}{qB} \approx \frac{mv}{qB} \quad (v \ll c)$$

$$f = \frac{v}{C} = \frac{v}{2\pi \rho} = \frac{v}{2\pi} \frac{qB}{mv}$$

$$= \frac{qB}{2\pi m} \quad \text{(constant!!)}$$

For a proton:

$$f_c = 15.2 \times B[T] \quad \text{MHz}$$

i.e. “RF” range

would not work for electrons!

“Cyclotron Frequency”

Accelerating “DEES”: by applying a voltage which oscillates at $f_c$, we can accelerate the particle a little bit each time around, allowing us to get to high energies with a relatively small voltage.
Round we go: the first cyclotrons

- ~1930 (Berkeley)
  - Lawrence and Livingston
  - K=80keV

- 1939 - 60” Cyclotron
  - Lawrence, et al. (LBL)
  - ~19 MeV (D₂)
  - Prototype for many
  - Parts are still in use!
Interlude: electrons vs. protons

- Electrons are point-like
  - Well-defined initial state
  - Full energy available to interaction

- Protons are made of quarks and gluons
  - Interaction take place between these constituents.
  - Only a small fraction of energy available, not well-defined.
  - Rest of particle fragments -> big mess!
As the trajectory of a charged particle is deflected, it emits “synchrotron radiation”

\[
\text{Radiated Power} \propto \frac{1}{\rho^2} \left( \frac{E}{m} \right)^4
\]

• **Protons:** Synchrotron radiation does not affect kinematics very much
  • Energy limited by strength of magnetic fields and size of ring

• **Electrons:** Synchrotron radiation dominates kinematics
  • To go higher energy, we have to *lower* the magnetic field and go to *huge* rings
  • Eventually, we lose the benefit of a circular accelerator, because we lose all the energy each time around.

Since the beginning, the energy frontier has belonged to proton (and/or antiproton) machines
Onward and upward!

- Two major advances allowed accelerators to go beyond the energies possible at cyclotrons
  - “Synchrotron” - in which the magnetic field is increased as the energy increases, such that particles continue to follow the same path.
    - Edward McMillan, 1945
  - “Strong focusing” - a technique in which magnetic gradients (non-uniform fields) are used to focus particles and keep them in a smaller beam pipe than was possible with cyclotrons.
    - Courant, Livingston and Snyder, 1952*

*actually invented in 1949 by a Greek-American electrical engineer name Nicholas Christofilos, but it was completely ignored at the time!
Understanding beam motion

- A particle of unit charge in a magnetic field will move with a local radius of curvature.

\[ \rho = \frac{p}{eB} \]

Field changes with location

- No matter how complex the magnetic fields are, if they are all scaled proportionally to the increasing momentum, particles will continue to follow the same trajectory as they accelerate.
  - Accelerators in which the magnetic fields are scaled this way are called “synchrotrons”

Summer Intern Talk, June 29, 2017
E. Prebys: Particle Accelerators
A positive particle coming out of the page off center in the horizontal plane will experience a *restoring* “kick”, *proportional to the displacement*.

Just like a “thin lens” in classical optics.
What about the other plane?

Pairs give net focusing in both planes -> “FODO cell”

The fundamental building block of synchrotrons and beam lines!

Luckily, if we place equal and opposite pairs of lenses, there will be a net focusing regardless of the order.
Example of FODO cells

“beta function”: proportional to square of beam size. Size alternates between the two planes.

150 meter Straight (half) - Mirror symmetric lattice

2 x

mirror point

13.5 x FODO cells

Arc matching cells

Alternating gradient quadrupoles
Strong focusing was originally implemented by building magnets with non-parallel pole faces to introduce a linear magnetic gradient = “combined function”, which both bent and focused the beam.

\[ B_y(x) = B_0 + B'x = \]

Later synchrotrons were built *separate* dipole and quadrupole magnets.
Example: Fermilab Main Ring

- First “separated function” lattice
- 1 km in radius
- First accelerated protons from 8 to 400 GeV in 1972

Basic Main Ring “FODO Cell” building block

Quadrupoles with alternating polarity (QF, QD) focus beam

Dipoles (B) bend beam into circle

They thought this idea was so cool that it needed its own logo
**Cyclotron:** Magnetic field stays fixed. Orbital radius increases as particles accelerate.
- **Pros:**
  - Inexpensive
  - Very high current
- **Cons:**
  - Limited energy
  - Weak focusing (larger beam sizes)

**Synchrotron:** All magnetic fields scale with momentum to keep particles in the same orbit
- **Pros:**
  - High energy
  - Strong focusing (precise beam control)
- **Cons:**
  - Cost
  - Cycle time limits average beam current

**But how do we accelerate beam?**
We will generally accelerate particles using structures that generate time-varying electric fields (RF cavities), either in a linear arrangement ("linac") or located within a circulating ring.

In both cases, we want to "phase" the RF so a nominal arriving particle will see the same accelerating voltage and therefore get the same boost in energy.
Examples of accelerating RF structures

Use resonant structures to make efficient use of power

Fermilab Drift Tube Linac (200MHz): oscillating field uniform along length

ILC prototype elliptical cell “π-cavity” (1.3 GHz): field alternates with each cell

37->53MHz Fermilab Booster cavity

Biased ferrite frequency tuner

E. Prebys: Particle Accelerators
Oscillating fields are timed ("phased") so that the accelerating electric field is always pointing in the right direction whenever a bunch passes through...
Multi-stage acceleration

- Early synchrotrons had low energy injection and provided all the acceleration in a single stage.
- The energy range of a single synchrotron is limited by:
  - Beams get smaller as they accelerate, so an aperture large enough for the injected beam is unreasonably large at high field.
  - Hysteresis effects result in excessive nonlinear terms at low energy (very important for colliders)
- Typical range 10-20 for colliders, larger for fixed target:
  - Fermilab Main Ring: 8-400 GeV (50x)
  - Fermilab Tevatron: 150-980 GeV (6.5x)
  - LHC: 400-7000 GeV (17x)
- The highest energy beams require multiple stages of acceleration, with high reliability at each stage.
Typically 10s of keV and mAs to 10s of mA of current. Want to accelerate as fast as possible before space charge blows up the beam!

FNAL H- source. Mix Cesium with Hydrogen to add electron.
Example: Fermilab complex today

Fermilab Accelerator Complex

- **Main Injector**: 120 GeV
- **Recycler Ring**: Fixed 8 GeV ring. Manipulates Booster beam
- ** Booster**: 8 GeV
- **Low-Energy Neutrino Experiments**: 8 GeV
- **High-Energy Neutrino Experiments**: 120 GeV
- **Linac**: 400 MeV
- **Ion Source**: 750 keV
- **Muon Experiments**: 8 GeV
- **Fixed-Target Experiments, Test Beam Facility**: 120 GeV + secondary beams

E. Prebys: Particle Accelerators

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Bunch/beam intensity are measured using inductive torioids.

Beam position is typically measured with beam position monitors (BPM’s), which measure the induced signal on opposing pickups.

Longitudinal profiles can be measured by introducing a resistor to measure the induced image current on the beam pipe -> Resistive Wall Monitor (RWM).
Beam profiles in beam lines can be measured using secondary emission multiwires (MW’s).

Can measure beam profiles in a circulating beam with a “flying wire scanner”, which quickly passes a wire through and measures signal vs time to get profile.

Non-destructive measurements include:

- Ionization profile monitor (IPM): drift electrons or ions generated by beam passing through residual gas
- Synchrotron light
  - Standard in electron machines
  - Also works in LHC

Beam profiles in MiniBooNE beam line

Flying wire signal in LHC
Colliding beams

- Two cars hitting each other at 60 mph...

- ...is about the same as one car going 120 mph hitting a parked car.

- But things get very different as we approach the speed of light...
The case for colliding beams

- For beam hitting a stationary proton, the “center of mass energy” (i.e. energy available to the reaction) is

\[ E_{CM} = \sqrt{2E_{beam}m_{target}c^2} \]

- On the other hand, for colliding beams (of equal mass and energy) it’s

\[ E_{CM} = 2E_{beam} \]

- To get the 14 TeV CM design energy of the LHC with a single beam on a fixed target would require that beam to have an energy of 100,000 TeV!

- \textbf{Would require a ring 10 times the diameter of the Earth!!}

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E. Prebys: Particle Accelerators

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First $e^+e^-$ collider

- ADA (Anello Di Accumulazione) at INFN, Frascati, Italy (1961)
  - 250 MeV $e^+$ x 250 MeV $e^-$

- It’s easier to collide $e^+e^-$, because synchrotron radiation naturally “cools” the beam to smaller size.
Summary: Evolution of the energy frontier

Available CM Energy (TeV)

- Fixed Target
- Colliding Beams

~a factor of 10 every 15 years

100
10
1
0.1
0.01
0.001

Year

1950 1970 1990 2010

BNL Cosmotron (1953)
Berkeley Bevatron (1954)
CERN PS (1959)
BNLAGS (1960)
FNAL Main Ring (1972) never at the energy frontier
CERN ISR (1971) First Hadron Collider!
FNAL Tevatron (1985) First Superconducting Synchrotron
CERN LHC (2010)
CERN LHC (design)

proton mass

E. Prebys: Particle Accelerators
Summer Intern Talk, June 29, 2017
Tunnel originally dug for LEP
- Built in 1980’s as an electron positron collider
- Max 100 GeV/beam, but 27 km in circumference!!
Design:
- 7 TeV+7 TeV proton beams
  - Can’t make enough antiprotons for the LHC
  - Magnets have two beam pipes, one going in each direction.
- Stored beam energy 150 times more than Tevatron
  - Each beam has only $5 \times 10^{-10}$ grams of protons, but has the energy of a train going 100 mph!!
- These beams are focused to a size smaller than a human hair to collide with each other!

- 27 km in circumference
- 2 major collision regions: CMS and ATLAS
- 2 “smaller” regions: ALICE and LHCb
Research machines: just the tip of the iceberg

Number of accelerators worldwide
\~ 26,000

- 41% Radiotherapy (>100,000 treatments/yr)*
- 44% Medical Radioisotopes
- 9% Research (incl. biomedical)
- 4% Industrial Processing and Research
- 1% Ion Implanters & Surface Modification
- 1% >1 GeV for research

Annual growth is several percent
Sales >3.5 B$/yr
Value of treated good > 50 B$/yr **
A 1 GeV Linac loads 1.5E14 protons into a non-accelerating synchrotron ring.

These are fast extracted onto a Mercury target

This happens at 60 Hz -> 1.4 MW

Neutrons are used for biophysics, materials science, industry, etc...
Light sources: too many to count

- Put circulating electron beam through an “undulator” to create synchrotron radiation (typically X-ray)
- Many applications in biophysics, materials science, industry.
- New proposed machines will use very short bunches to create coherent light.
Other uses of accelerators

- Radioisotope production
- Medical treatment
- Electron welding
- Food sterilization
- Catalyzed polymerization
- Even art...

In a “Lichtenberg figure”, a low energy electron linac is used to implant a layer of charge in a sheet of lucite. This charge can remain for weeks until it is discharged by a mechanical disruption.
Thank you for your attention!
The front end of any modern hadron accelerator looks something like this (Fermilab front end)

From here, particles go to a “Linac” (linear accelerator)...
Compare Fermilab LINAC (K=400 MeV) to LHC (K=7000 GeV)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Equation</th>
<th>Injection</th>
<th>Extraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>proton mass</td>
<td>m [GeV/c^2]</td>
<td></td>
<td></td>
<td>0.938</td>
</tr>
<tr>
<td>kinetic energy</td>
<td>K [GeV]</td>
<td></td>
<td>.4</td>
<td>7000</td>
</tr>
<tr>
<td>total energy</td>
<td>E [GeV]</td>
<td>$K + mc^2$</td>
<td>1.3382</td>
<td>7000.938</td>
</tr>
<tr>
<td>momentum</td>
<td>p [GeV/c]</td>
<td>$\sqrt{E^2 - (mc^2)^2}$</td>
<td>0.95426</td>
<td>7000.938</td>
</tr>
<tr>
<td>rel. beta</td>
<td>$\beta$</td>
<td>$(pc)/E$</td>
<td>0.713</td>
<td>0.9999999991</td>
</tr>
<tr>
<td>rel. gamma</td>
<td>$\gamma$</td>
<td>$E/(mc^2)$</td>
<td>1.426</td>
<td>7461.5</td>
</tr>
<tr>
<td>beta-gamma</td>
<td>$\beta\gamma$</td>
<td>$(pc)/(mc^2)$</td>
<td>1.017</td>
<td>7461.5</td>
</tr>
<tr>
<td>rigidity</td>
<td>$(B\rho)$ [T-m]</td>
<td>$p[GeV]/(0.2997)$</td>
<td>3.18</td>
<td>23353</td>
</tr>
</tbody>
</table>

This would be the radius of curvature in a 1 T magnetic field or the field in Tesla needed to give a 1 m radius of curvature.
The relationship between angle (in Radians) and the fundamental trigonometric functions is

For very small angles (θ<<1)

\[ y \approx s, x \approx r \rightarrow \sin \theta \approx \tan \theta \approx \theta \]

This is known as the “paraxial approximation”, and it will be very important for us.
If the path length through a transverse magnetic field is short compared to the bend radius of the particle, then we can think of the particle receiving a transverse “kick”

\[ p_\perp \approx F_\perp t = qvBt = qvB(l/v) = qBl \]

and it will be bent through small angle

“paraxial approximation”

In this “thin lens approximation”, a dipole is the equivalent of a prism in classical optics.
A positive particle coming out of the page off center in the horizontal plane will experience a *restoring kick proportional to the displacement*

\[ \Delta \theta \approx -\frac{B_y l}{(B \rho)} = -\frac{B'lx}{(B \rho)} \]

*or linear term in a gradient magnet*
Some important early synchrotrons

- **Berkeley Bevatron,**
  - 1954 (weak focusing)
  - 6.2 GeV protons
  - Discovered antiproton

- **CERN Proton Synchrotron (PS),**
  - 1959
  - 628 m circumference
  - 28 GeV protons
  - Still used in LHC injector chain!

- **CERN Proton Synchrotron (PS),**
  - 1960
  - 808 m circumference
  - 33 GeV protons
  - Discovered charm quark, CP violation, muon neutrino
Injection and Extraction

- We typically would like to extract (or inject) beam by switching a magnetic field on between two bunches (order ~10-100 ns)

- Unfortunately, getting the required field in such a short time would result in prohibitively high inductive voltages, so we usually do it in two steps:
  - fast, weak “kicker”
  - slower (or DC) extraction magnet with zero field on beam path.
Some other important accelerators (past):

**LEP (at CERN):**
- 27 km in circumference
- $e^+e^-$
- Primarily at $2E = M_Z$ (90 GeV)
- Pushed to $E_{CM} = 200$ GeV
- $L = 2E31$
- Highest energy *circular* $e^+e^-$ collider that will ever be built.
- Tunnel now houses LHC

**SLC (at SLAC):**
- 2 km long LINAC accelerated electrons AND positrons on opposite phases.
- $2E = M_Z$ (90 GeV)
- polarized
- $L = 3E30$
- Proof of principle for linear collider
B-Factories

- B-Factories collide $e^+e^-$ at $E_{\text{CM}} = M(\Upsilon(4S))$.
- Asymmetric beam energy (moving center of mass) allows for time-dependent measurement of B-decays to study CP violation.

**KEKB (Belle Experiment):**
- Located at KEK (Japan)
- 8GeV e- x 3.5 GeV e+
- Peak luminosity $>1\times10^{34}$

**PEP-II (BaBar Experiment):**
- Located at SLAC (USA)
- 9GeV e- x 3.1 GeV e+
- Peak luminosity $>1\times10^{34}$
Relativistic Heavy Ion Collider (RHIC)

- Located at Brookhaven:
- Can collide protons (at 28.1 GeV) and many types of ions up to Gold (at 11 GeV/amu).
- Luminosity: 2E26 for Gold
- Goal: heavy ion physics, quark-gluon plasma, ??
Locate at Jefferson Laboratory, Newport News, VA
6GeV e- at 200 uA continuous current
Nuclear physics, precision spectroscopy, etc
Colliding Beam Luminosity

Circulating beams typically “bunched” (number of interactions)

\[
L = \left( \frac{N_1 N_2}{A} \right) r_b = \left( \frac{N_1 N_2}{A} \right) n \frac{c}{C}
\]

Cross-sectional area of beam

Circumference of machine

Number of bunches

crossing rate

E. Prebys: Particle Accelerators  
Summer Intern Talk, June 29, 2017
*Kate McAlpine (http://www.youtube.com/user/alpinekat)
“Compact” (ha ha) Linear Collider (CLIC)?

- Use low energy, high current electron beams to drive high energy accelerating structures

- Up to 1.5 x 1.5 TeV, but VERY, VERY hard
The energy of Hadron colliders is limited by feasible size and magnet technology. Options:

- Get very large (~100 km circumference)
- More powerful magnets (requires new technology)
Future Circular Collider (FCC)

- Currently being discussed for ~2030s
- 80-100 km in circumference
- Niobium-3-Tin (Nb$_3$Sn) magnets.
- ~100 TeV center of mass energy (~7 x LHC)
Leptons vs. Hadrons revisited

- Because 100% of the beam energy is available to the reaction, a lepton collider is competitive with a hadron collider of ~5-10 times the beam energy (depending on the physics).
- A lepton collider of >1 TeV/beam could compete with the discovery potential of the LHC
  - A lower energy lepton collider could be very useful for precision tests, but I’m talking about direct energy frontier discoveries.
- Unfortunately, building such a collider is VERY, VERY hard
  - Eventually, circular e⁺e⁻ colliders will radiate away all of their energy each turn
    - LEP reached 100 GeV/beam with a 27 km circumference synchrotron!
  - Next discovery e⁺e⁻ collider will be linear
LEP was the limit of circular $e^+e^-$ colliders

- Next step must be linear collider
- Proposed ILC 30 km long, 250 x 250 GeV $e^+e^-$ (NOT energy frontier)

We don’t yet know whether that’s high enough energy to be interesting

- Need to wait for LHC results
- What if we need more?
Muon colliders?

- Muons are pointlike, like electrons, but because they’re heavier, synchrotron radiation is much less of a problem.
- Unfortunately, muons are unstable, so you have to produce them, cool them, and collide them, before they decay.
Many advances have been made in exploiting the huge fields that are produced in plasma oscillations.

Potential for accelerating gradients many orders of magnitude beyond RF cavities.

Still a long way to go for a practical accelerator.
A body in motion will have a total energy given by
\[
E = \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \equiv \gamma mc^2
\]

The difference between this and \( mc^2 \) is called the “kinetic energy”

Here are some examples of kinetic energy

<table>
<thead>
<tr>
<th>Example</th>
<th>Velocity</th>
<th>Velocity/ Speed of light</th>
<th>Kinetic Energy/(mc²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Race car</td>
<td>150 mph</td>
<td>.0000002</td>
<td>.0000000000000025</td>
</tr>
<tr>
<td>Apollo 12 (fastest people)</td>
<td>24,791 mph</td>
<td>.000037</td>
<td>.0000000000068</td>
</tr>
<tr>
<td>Fermilab LINAC (K=400 MeV)</td>
<td>214,000,000 m/s</td>
<td>.71</td>
<td>.43</td>
</tr>
<tr>
<td>Proton in the LHC (full energy)</td>
<td>Light minus 2.7 m/s</td>
<td>.9999999991</td>
<td>7500</td>
</tr>
<tr>
<td>Electron in LEP</td>
<td>Light minus 3.6 mm/s</td>
<td>.999999999988</td>
<td>203,000</td>
</tr>
</tbody>
</table>
Rough (and not particularly accurate) outline

- Understanding energy scales
- Motivation for particle accelerators
- Getting to higher energy
  - Cyclotrons
  - Strong magnetic focusing
  - Acceleration
  - Colliding beams
- Superconducting magnets
- State of the art
- Other uses for accelerators
You’ve all learned (I hope) that like charges repel and opposite charges attract.

This can be interpreted as one charge creating an “electric field” which accelerates the other charge.

The formula for force due to an electric field is:

\[ \vec{F} = q \vec{E} = m \frac{\Delta v}{\Delta t} = m \vec{a} \]

- \( \vec{F} \): Force [Newtons]
- \( q \): Charge [Coulombs]
- \( \vec{E} \): “Electric Field” [Newtons/Coulomb] or [Volts/Meter]
- \( m \): Mass [kg]
- \( \Delta v \): Change in velocity
- \( \Delta t \): Change in time
- \( a \): Acceleration [m/s²]

\[ E = \frac{kQ}{r^2} \]

\( k \) is the Coulomb constant.

*we’ll get to magnetic fields shortly*
“Work” is force over a distance.

\[ W [\text{Joules}] = F [\text{Newtons}] \cdot \Delta x [\text{meters}] \]

For an electric field, this becomes

\[ W = F \cdot \Delta x = qE \cdot \Delta x = q(E \cdot \Delta x) \equiv q(\Delta V [\text{Joules/Coulomb} = \text{Volts}]) \]

Electric field over distance = “voltage change”

In practice, our sources of electric fields generally involve chemical or mechanical work, so we usually turn this relationship around. For example, consider a plate capacitor:

The voltage of the source produces charge distributions on the two plates, resulting in a uniform field

\[ E = \frac{V}{d} \]

The force felt by a charged particle between the plates will be

\[ F = qE = q \frac{V}{d} \]

And the total energy gained by a particle crossing the gap will be

\[ W = Fd = qEd = q \frac{V}{d} d = qV \]

Which leads us to...
Glossary: standard symbols

- I will use a lot of standard symbols, some of which make sense:
  - E: electric field or total energy
  - K: kinetic energy
  - m: mass
  - v: velocity
  - V: voltage
  - P: power
  - e: fundamental electric charge

- some of which don’t
  - p: momentum
  - c: speed of light
  - q: charge
  - I: current
  - B: magnetic field

- and some of which are Greek
  - ρ (“rho”): curvature radius
  - θ (“theta”): angle
  - γ (“Lorentz gamma”): “time dilation factor”
You might think you understand how magnets work, but the usual magnetic forces you’re familiar with are actually really complicated, involving the motion of electrons inside matter.

At their most basic, magnetic forces are exerted by moving charges (currents) on other moving charges.

As we did with electric force, we’ll look at this as one current creating a “magnetic field” and the other experiencing it.
Understanding the force between two currents

One current produces the field

The other one experiences it
Cyclotrons rely on the fact that magnetic fields between two pole faces are never perfectly uniform. This prevents the particles from spiraling out of the pole gap. In early synchrotrons, radial field profiles were optimized to take advantage of this effect, but in any weak focused beams, the beam size grows with energy.

The most famous weak focusing accelerator was the Berkeley Bevatron, which had a kinetic energy of 6.2 GeV - High enough to make antiproton (and win a Nobel Prize) - It had an aperture 12”x48”!
First proton collider: CERN Intersecting Storage Rings (ISR)

- 1971
- 31 GeV + 31 GeV colliding proton beams.
  - Highest CM Energy for 10 years
- Set a luminosity record that was not broken for 28 years!
Protons from the SPS were used to produce antiprotons, which were collected. These were injected in the opposite direction (same beam pipe) and accelerated. First collisions in 1981. Discovery of W and Z in 1983. Nobel Prize for Rubbia and Van der Meer.

- Energy initially 270+270 GeV
- Raised to 315+315 GeV
- Limited by power loss in magnets!
The maximum Sp̅S energy was limited by the maximum power loss that the conventional magnets could support. 
- LHC made out of such magnets would be roughly the size of Rhode Island!

- Highest energy colliders only possible using superconducting magnets
- Must take the bad with the good
  - Conventional magnets are simple and naturally dissipate energy as they operate
  - Superconducting magnets are complex and represent a great deal of stored energy which must be handled if something goes wrong

\[ E \propto B^2 \]
When is a superconductor not a superconductor?

- Superconductor can change phase back to normal conductor by crossing the “critical surface”

- When this happens, the conductor heats quickly, causing the surrounding conductor to go normal and dumping lots of heat into the liquid Helium “quench”
  - all of the energy stored in the magnet must be dissipated in some way

- Dealing with quenches is the single biggest issue for any superconducting synchrotron!
Quench example: MRI magnet*

*pulled off the web. We recover our Helium.
The Fermilab Main Ring was completed in 1972 with normal magnets.

By the late 70s, serious plans began for a superconducting collider in the same tunnel, followed by construction.

- Dubbed “Saver Doubler” (later “Tevatron”)
- Helen Edwards led the construction effort.

- 1985 - First proton-antiproton collisions in Tevatron
  - Most powerful accelerator in the world for the next quarter century

- 1995 - Top quark discovery

- 2011 - Tevatron shut down after successful LHC startup
The two axioms of Special Relativity

- The laws of physics are the same in any “inertial” (non-accelerating) frame of reference
- The measured speed of light in a vacuum is the same in every inertial frame of reference

Example

Observer 1 is inside a spaceship. He shines a light to a mirror on the other side and measures the time ($T_1$) it takes to return.

Observer 2 is watching the spaceship go by. In his “frame”, the light travels a longer distance – at the same velocity ➔ the time he measures ($T_2$) is longer!

*See SMP lecture by Mehreen Sultana, March 11, 2017*
If the speed of light is the same in both frames, then the measured time must be different!

\[ cT_1 = 2L \]
\[ \rightarrow L = \frac{1}{2} cT_1 \]

Substitute L and square both sides

\[ cT_2 = 2D \]
\[ = 2 \sqrt{L^2 + \left(\frac{1}{2} vT_2\right)^2} \]

\[ c^2 T_2^2 = c^2 T_1^2 + v^2 T_2^2 \]
\[ \rightarrow \left(1 - \frac{v^2}{c^2}\right) T_2 = T_1 \]
\[ \rightarrow T_2 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} T_1 \equiv \gamma T_1 \]

Lorentz “time dilation” factor (“gamma”)

We’re going to see a lot of this factor. We’ll discuss the behavior shortly.
In case you didn’t believe me...

Me

“Buck’s River Road Exxon”

The guy who hired me: Prof. Kwan Lai

Fermilab E-706 Rochester Group
~1987

Then

Now
Summary: Units

- We will use (mostly) SI units:
  - Length: m (mm, \( \mu \text{m} \), etc)
  - Mass: kg
  - Time: s (\( \mu \text{s} \), ns, etc)
  - Electrical units: Volts, Amps, etc

- The exceptions be energy, mass, and momentum
  - For energy, we will use electron-Volts [eV]
  - For mass, we will use “equivalent energy” [eV/c^2]
    - Example: proton
      
      \[
      m_p c^2 = 938 \text{ MeV} \quad \rightarrow \quad m_p = 938 \text{ MeV/c}^2
      \]
    - We will also use equivalent energy for momentum [eV/c]
      
      \[
      p \text{ [eV/c]} = \frac{\gamma mv \text{ [kg-m/s]}}{\text{electron charge (1.6} \times 10^{-19} \text{C)}}
      \]
  - This looks weird, but it works very well with the relationship

\[
E^2 = (mc^2)^2 + (pc)^2
\]
So why don’t we stick to electrons??
Initial acceleration

Old: Static

Static acceleration from Cockcroft-Walton.
FNAL = 750 keV
max ~1 MeV

New: RF Quadrupole (RFQ)

RF structure combines an electric focusing quadrupole with a longitudinal accelerating gradient.
Because the velocity is changing quickly, the first linac is generally a Drift Tube Linac (DTL), also known as an “Alvarez Linac”, which can match the drift tube separation to the changing velocity.

As energy gets higher, switch to “pi-cavities”, which are more efficient.

After that, beam goes to a series of progressively larger synchrotron rings.