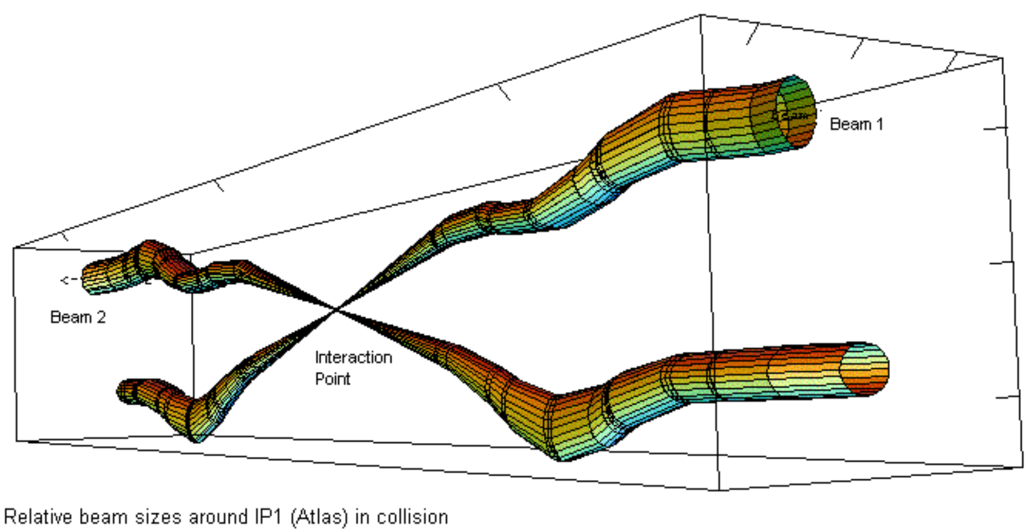
HCPSS 2014
Hadron Collider Physics Summer School

August 11 - 22, 2014 Fermi National Accelerator Laboratory

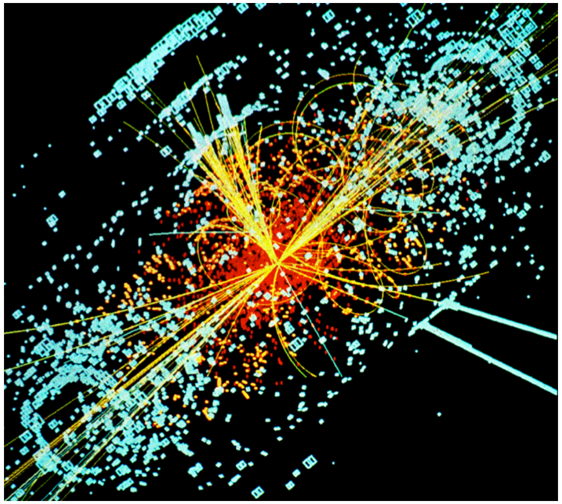
FERMILAB-CERN HADRON COLLIDER PHYSICS Summer School 2014

Hadron Colliders

Eric Prebys, FNAL



LHC Interaction Region



Lecture 1

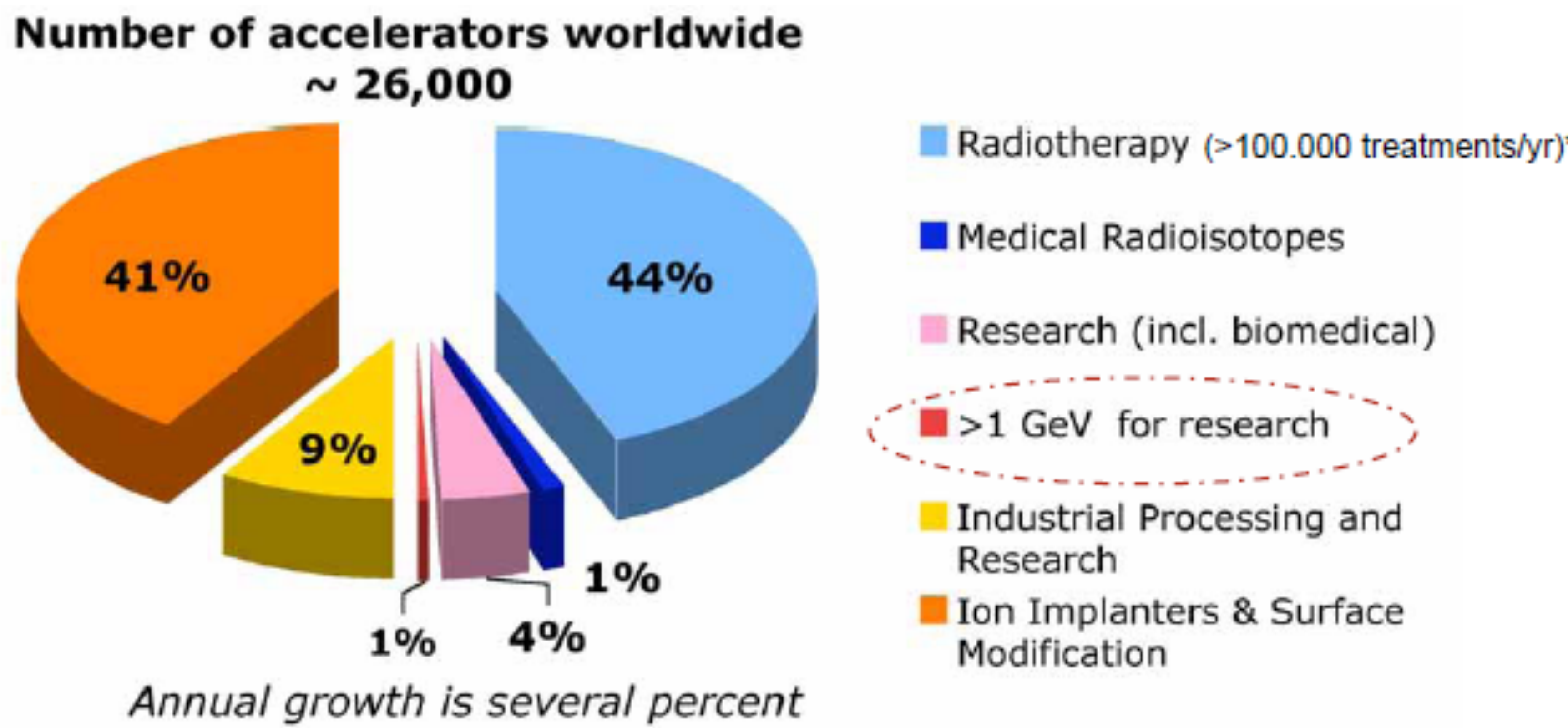
Comments

- These lectures will focus on the underlying physics and evolution of the highest energy hadron accelerators
  - This has largely driven the development of the technology; *however*
  - High energy research machines are a tiny fraction (~1%) of the particle accelerators in use today.
- I'll be fairly rigorous
  - In the end, you should have a reasonably quantitative understanding of most of the accelerator jargon you'll hear in a typical high energy physics talk:
    - "Lattice"
    - "Beta function"
    - "Tune" and "Tune shift"
    - "Emittance"
    - "RF"
    - "Luminosity"
    - "Squeeze"
    - etc.

Outline of Lectures

- Accelerator physics basics
  - Transverse motion
  - Longitudinal motion
  - Colliding beams
- LHC specific topics
  - Maximizing luminosity
  - Upgrade plans
- Special topics
  - Tricks of the trade
  - Instrumentation
  - etc

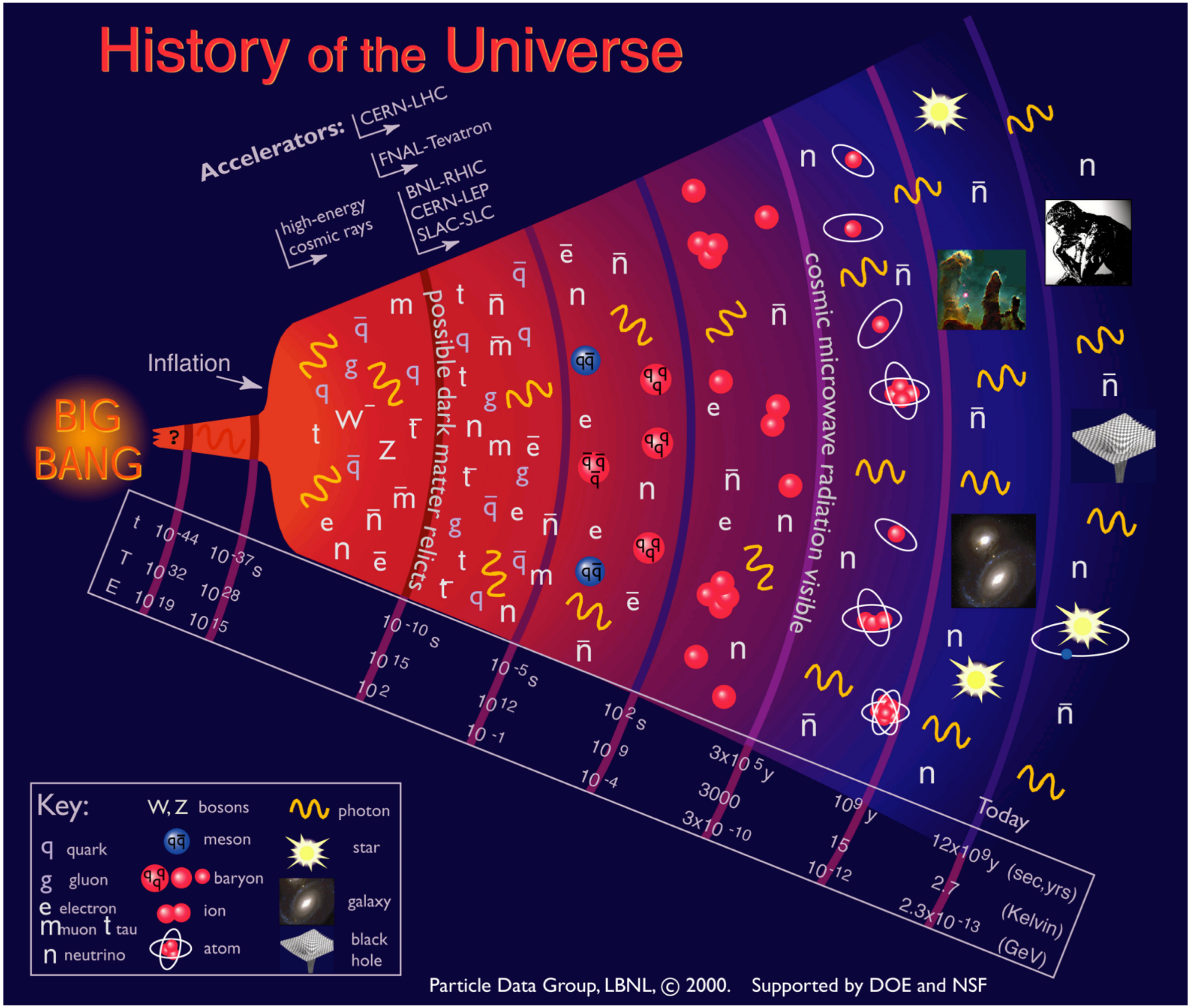
Some Perspective: Just the Tip of the Iceberg



Sales >3.5 B$/yr

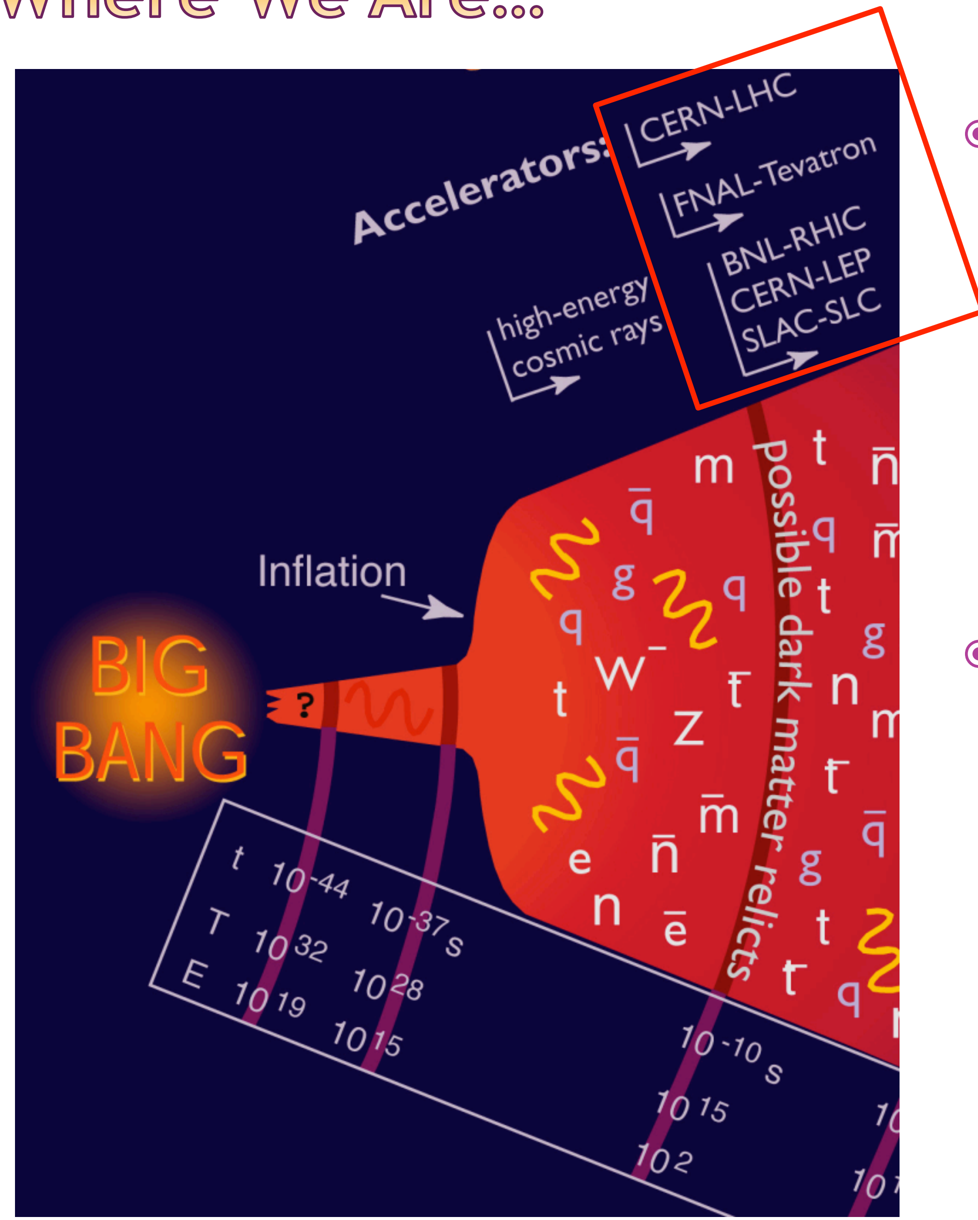
Value of treated good > 50 B$/yr **

Motivation



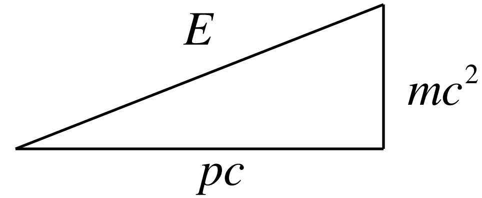
Going to higher energies = going back in time

Where We Are…



- Accelerators allow us to go back 13.8 *billion years* and recreate conditions that existed a *few trillionths of a second* after the Big Bang
  - the place where our current understanding of physics breaks down.
- In addition to high energy, we need high
  "luminosity"
  that is, lots of particles interacting, to see rare processes.

Relativity and Units



- Basic Relativity

$$\beta \equiv \frac{v}{c}$$

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

$$\text{momentum } p = \gamma m v$$

$$\text{total energy } E = \gamma m c^2$$

$$\text{kinetic energy } K = E - mc^2$$

$$E^2 = \sqrt{\left(mc^2\right)^2 + \left(pc\right)^2}$$

Some Handy Relationships

$$\beta = \frac{pc}{E}$$

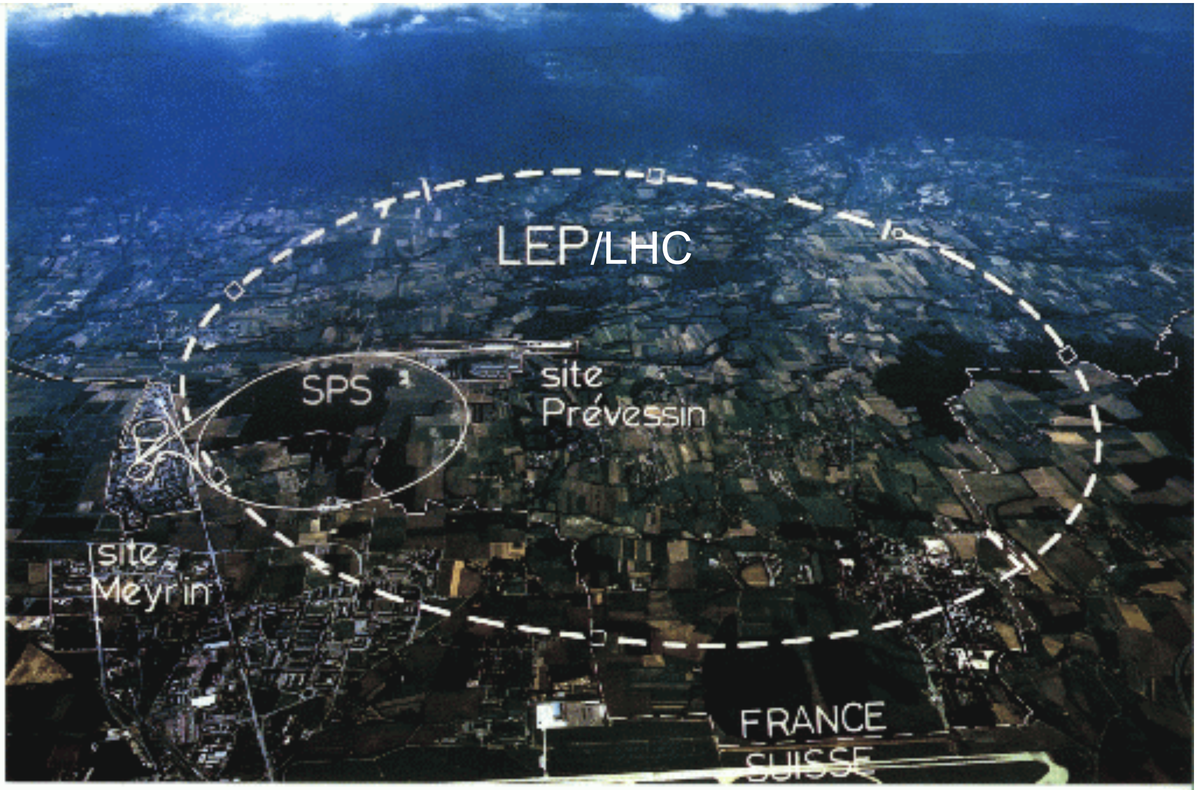
$$\gamma = \frac{E}{mc^2}$$

$$\beta\gamma = \frac{pc}{mc^2}$$

These units make these relationships really easy to calculate

- Units
  - For the most part, we will use SI units, except
    - Energy: eV (keV, MeV, etc) [1 eV = 1.6×10^{-19} J]
    - Mass: eV/c^2 [proton = 1.67×10^{-27} kg = 938 MeV/c^2]
    - Momentum: eV/c [proton @ β=.9 = 1.94 GeV/c]
  - In the US and Europe, we normally talk about the kinetic energy (*K*) of a particle beam, although we'll see that momentum really makes more sense.

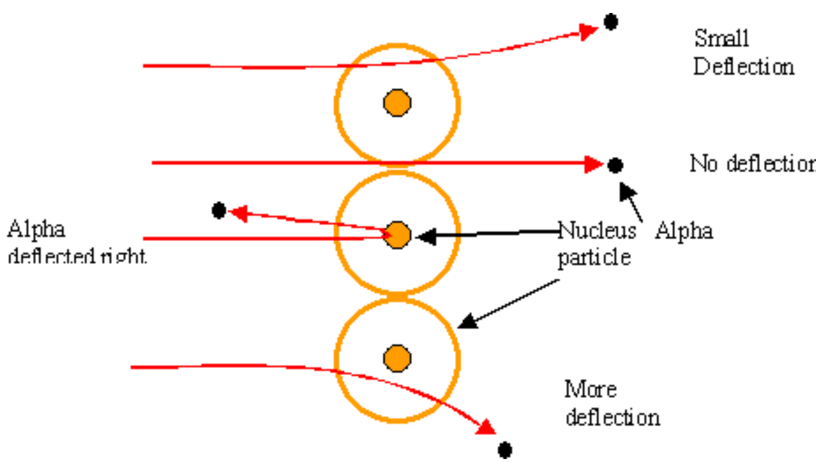
State of the Art: Large Hadron Collider (LHC)



- Built at CERN, straddling the French/Swiss border
- 27 km in circumference
- Has collided two proton beams at 4000 GeV each
- In 2015, will reach (almost) design energy of 7000 GeV/beam.
- That's where we are. Now let's see how we got here…

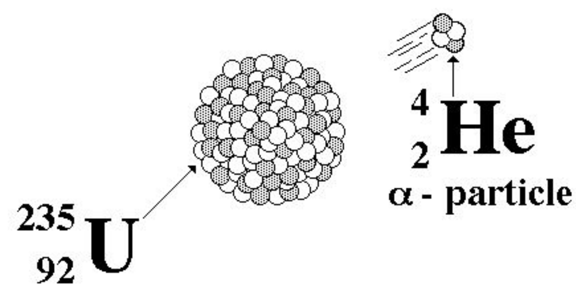
Rewind: Some Pre-History

- The first artificial acceleration of particles was done using "Crookes tubes", in the latter half of the 19th century
  - These were used to produce the first X-rays (1875)
  - At the time no one understood what was going on
- The first "particle physics experiment" told Ernest Rutherford the structure of the atom (1911)



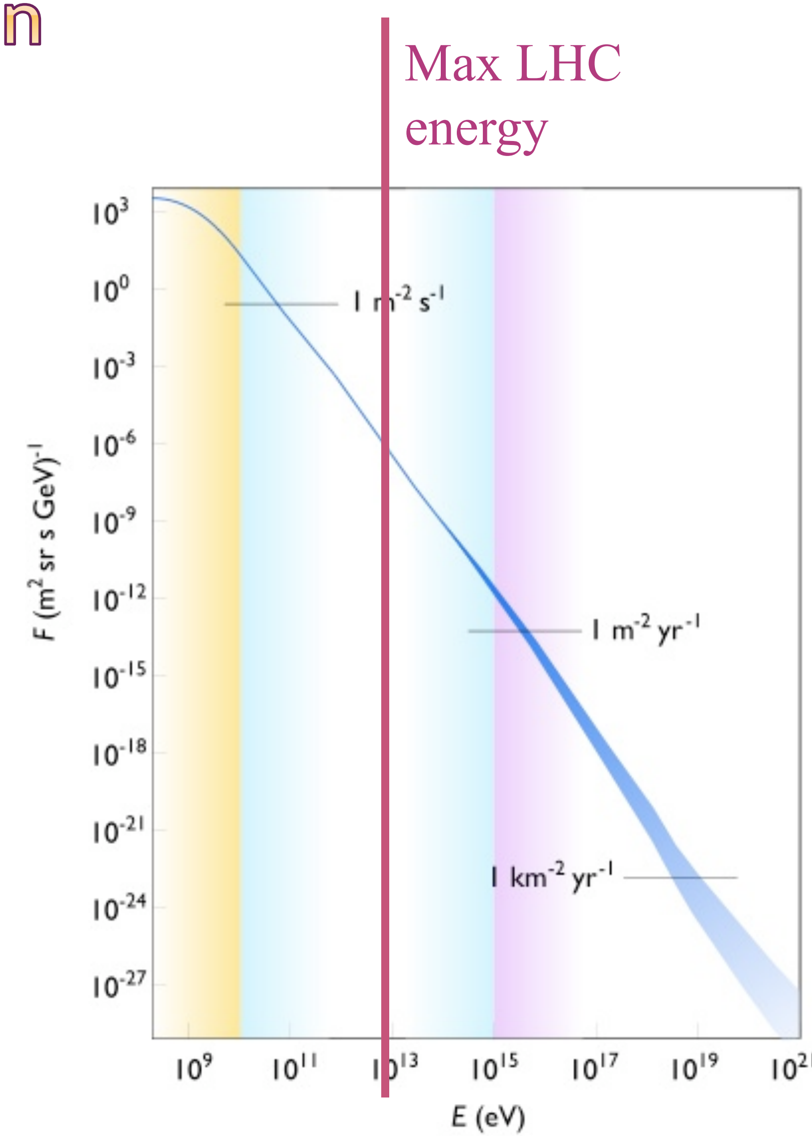
Study the way radioactive particles "scatter" off of atoms

- In this case, the "accelerator" was a naturally decaying ^{235}U nucleus



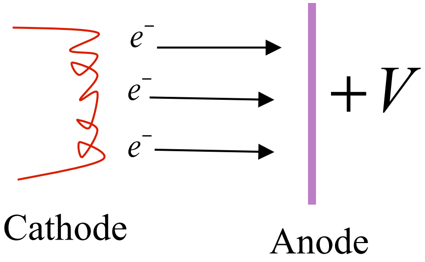
Natural Particle Acceleration

- Radioactive sources produce maximum energies of a few million electron volts (MeV)
- Cosmic rays reach energies of ~1,000,000,000 x LHC but the rates are too low to be useful as a study tool
  - Remember what I said about "luminosity".
- However, low energy cosmic rays are extremely useful for detector testing, commissioning, etc.



Man-made Particle Acceleration

The simplest accelerators accelerate charged particles through a *static* electric field. Example: **vacuum tubes** (or CRT TV's)



$$K = eEd = eV$$

Limited by magnitude of electric field:

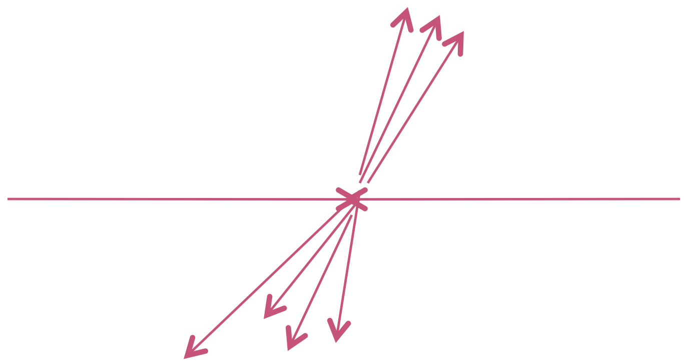
- CRT display ~keV
- X-ray tube ~10's of keV
- Van de Graaf ~MeVs

FNAL Cockroft-Walton = 750 kV

Solutions:

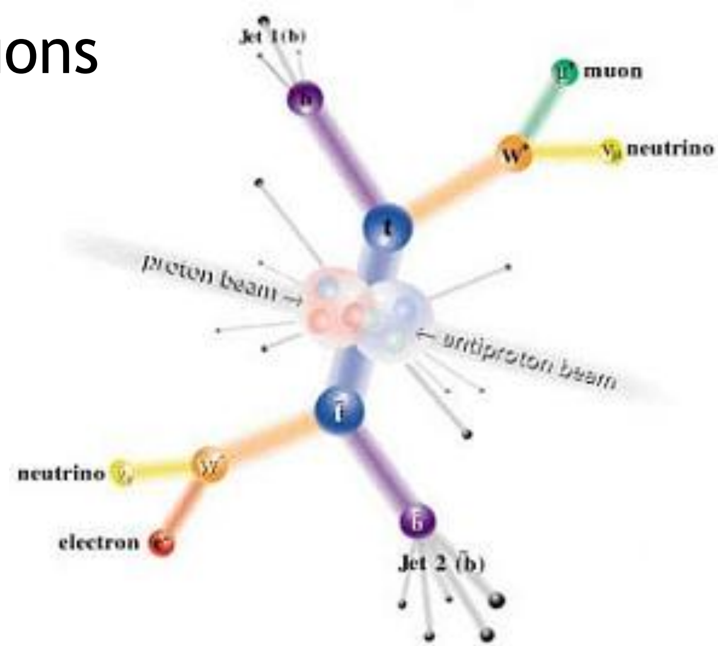
- Alternate fields to keep particles in accelerating fields -> Radio Frequency (RF) acceleration
- Bend particles so they see the same accelerating field over and over -> cyclotrons, synchrotrons

Interlude: Electrons vs. Protons



- Electrons are point-like
  - Well-defined initial state
  - Full energy available to interaction

- Protons are made of quarks and gluons
  - Interaction take place between these constituents.
  - Only a small fraction of energy available, not well-defined.
  - Rest of particle fragments -> big mess!



So why not stick to electrons?

Synchrotron Radiation

As the trajectory of a charged particle is deflected, it emits "synchrotron radiation"

$$\text{Radiated Power} \propto \frac{1}{\rho^2}\left(\frac{E}{m}\right)^4$$

Radius of curvature

An electron will radiate about *10^{13} times more power* than a proton of the same energy!!!!

- **Protons:** Synchrotron radiation does not affect kinematics very much
  - Energy limited by strength of magnetic fields and size of ring
- **Electrons:** Synchrotron radiation dominates kinematics
  - To to go higher energy, we have to *lower* the magnetic field and go to *huge* rings
  - Eventually, we lose the benefit of a circular accelerator, because we lose all the energy each time around.

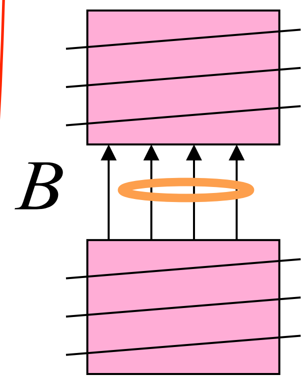
Since the beginning, the "energy frontier" has belonged to proton (and/or antiproton) machines, while electrons are used for precision studies and other purposes.

Now, back to the program…

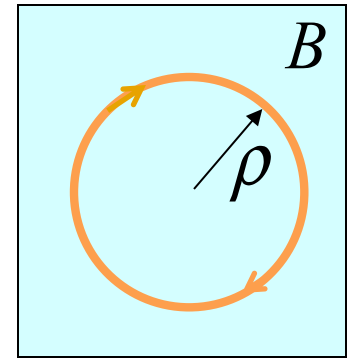
The Cyclotron (1930's)

would not work for electrons!

side view



top view



- A charged particle in a uniform magnetic field will follow a circular path of radius

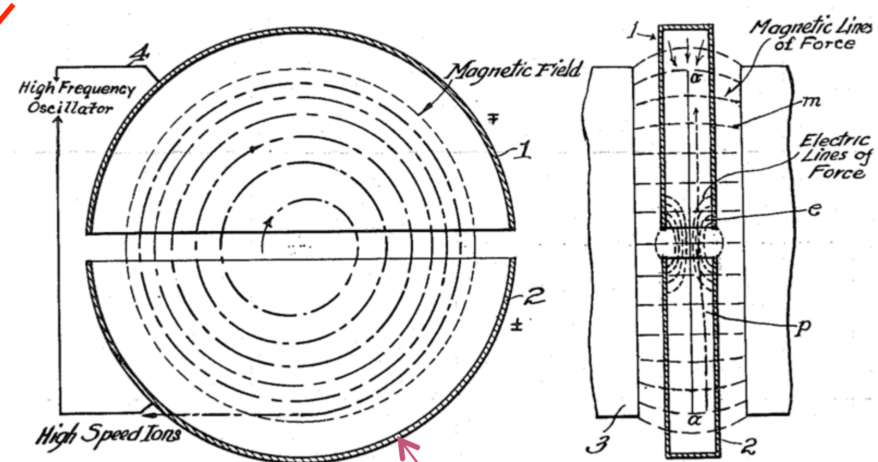
$$\rho = \frac{p}{qB} \approx \frac{mv}{qB} \quad (v \ll c)$$

$$f = \frac{v}{2\pi\rho}$$

$$= \frac{qB}{2\pi m} \quad \text{(constant!!)}$$

$$\Omega_s = \frac{f}{2\pi} = \frac{qB}{m}$$

"Cyclotron Frequency"



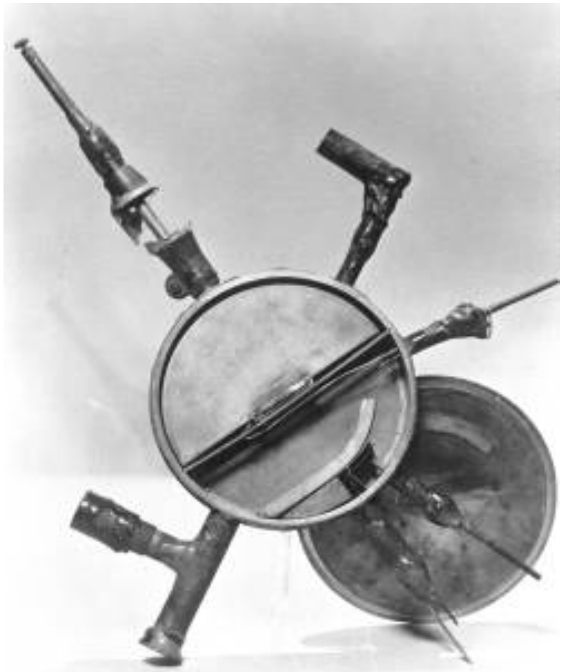
For a proton:

$$f_C = 15.2 \times B[T] \;\; \text{MHz}$$

i.e. "RF" range

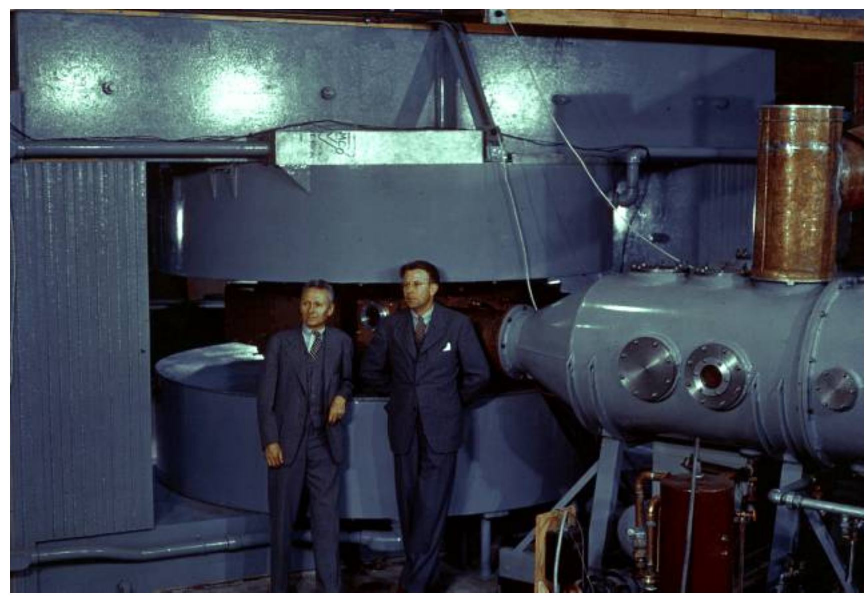
Accelerating "DEES": by applying a voltage which oscillates at f_c, we can accelerator the particle a little bit each time around, allowing us to get to high energies with a relatively small voltage.

Round and Round We Go: the First Cyclotrons



- ~1930 (Berkeley)
  - Lawrence and Livingston
  - K=80KeV
  - Fit in your hand

- 1935 - 60” Cyclotron
  - Lawrence, et al. (LBL)
  - ~19 MeV (D_2)
  - Prototype for many



Onward and Upward!

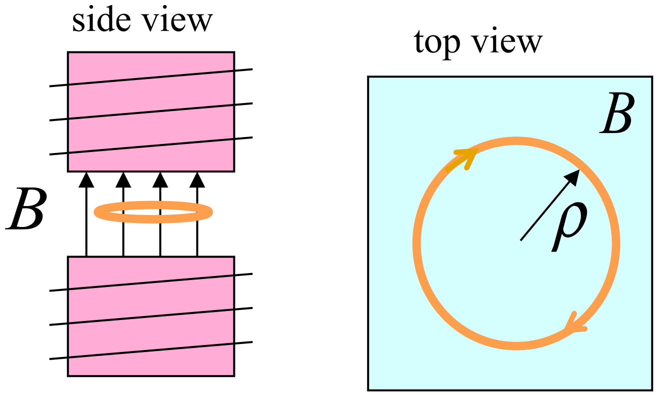
- Cyclotrons were limited by three problems
  - Constant frequency breaks down at ~10% speed of light
    - Solved with variable frequency "synchro-cyclotrons"
      ➔ phase stability (more about this later)
  - As energy goes up, magnet gets huge
  - Beams are not well focused and get larger with energy
- Two major advances allowed accelerators to go beyond the energies and intensities possible at cyclotrons
  - "Synchrotron" – in which the magnetic field is increased as the energy increases (proportional to momentum), such that particles continue to follow the same path. (McMillan, 1945)
  - "Strong focusing" – a technique in which magnetic gradients (non-uniform fields) are used to focus particles and keep them in a smaller beam pipe than was possible with cyclotrons. (Christofilos, 1949; Courant, Livingston, and Snyder, 1952)
- Note: still plenty of uses for cyclotrons (simple, inexpensive, rapid cycling)
  - Medical treatments
  - Isotope production
  - Nuclear physics

Understanding Beam Motion: Beam "rigidity"

- The relativistically correct form of Newton's Laws for a particle in an electromagnetic field is:

$$\vec{F} = \frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right); \quad \vec{p} = \gamma m \vec{v}$$

- A particle of unit charge in a uniform magnetic field will move in a circle of radius



$$\rho = \frac{p}{eB}$$

$$(B\rho) = \frac{p}{e}$$

constant for fixed energy!

T-m^2/s=V

$$(B\rho)c = \frac{pc}{e}$$

units of eV in our usual convention

Beam "rigidity" = constant at a given momentum (even when B=0!)

$$\boxed{(B\rho)[\text{T-m}] = \frac{p[\text{eV/c}]}{c[\text{m/s}]} \approx \frac{p[\text{MeV/c}]}{300}}$$

Remember forever!

If all magnetic fields are scaled with the momentum as particles accelerate, the trajectories remain the same

➔"synchrotron"

Example Beam Parameters

- Compare Fermilab LINAC (K=400 MeV) to LHC (K=7000 GeV)

Parameter	Symbol	Equation	Injection	Extraction
proton mass	m [GeV/c^2]		0.938	
kinetic energy	K [GeV]		.4	7000
total energy	E [GeV]	$K + mc^2$	1.3382	7000.938
momentum	p [GeV/c]	$\sqrt{E^2 - (mc^2)^2}$	0.95426	7000.938
rel. beta	β	$(pc)/E$	0.713	0.999999991
rel. gamma	γ	$E/(mc^2)$	1.426	7461.5
beta-gamma	βγ	$(pc)/(mc^2)$	1.017	7461.5
rigidity	(Bρ) [T-m]	$p[\mathrm{GeV}]/(.2997)$	3.18	23353.

This would be the radius of curvature in a 1 T magnetic field *or* the field in Tesla needed to give a 1 m radius of curvature.

Weak Focusing

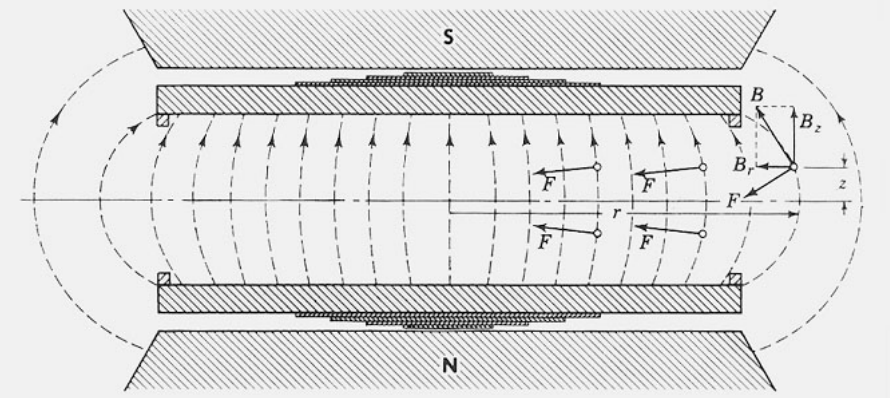
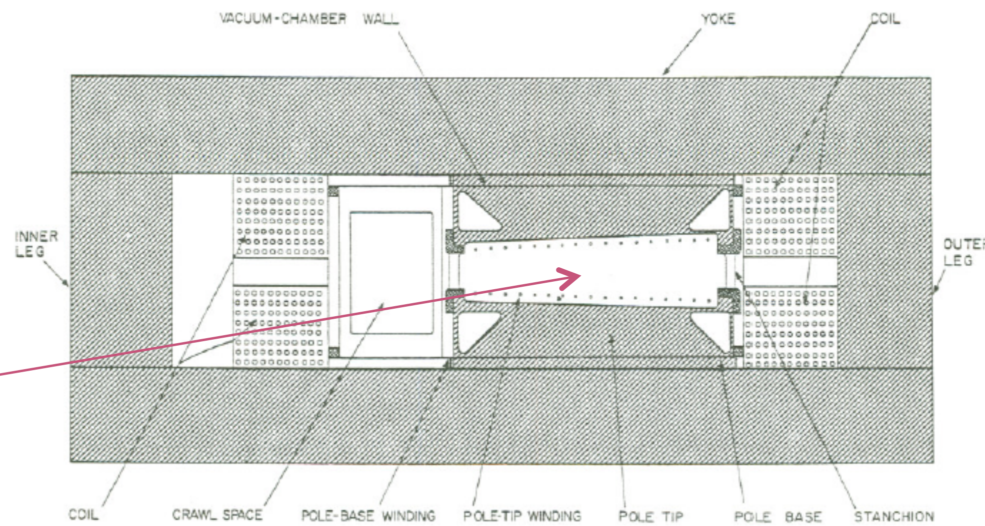


Fig. 6-7. Radially decreasing magnetic field between poles of a cyclotron magnet, showing shims for field correction.

- Cyclotrons relied on the fact that magnetic fields between two pole faces are never perfectly uniform.
- This prevents the particles from spiraling out of the pole gap.
- In early synchrotrons, radial field profiles were optimized to take advantage of this effect, but in any weak focused beams, *the beam size grows with energy.*
- The most famous weak focusing synchrotron was the Berkeley Bevatron, which had a kinetic energy of 6.2 GeV
  - High enough to make antiprotons (and win a Nobel Prize)
  - It had an aperture 12”x48”!

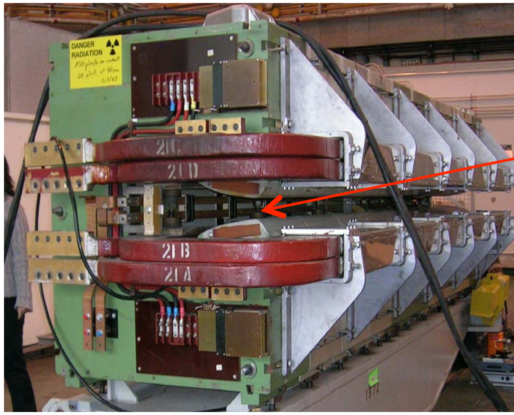


Strong Focusing

- Strong focusing utilizes alternating magnetic gradients to precisely control the focusing of a beam of particles
  - The principle was first developed in 1949 by Nicholas Christofilos, a Greek-American engineer, who was working for an elevator company in Athens at the time.
  - Rather than publish the idea, he applied for a patent, and it went largely ignored.
  - The idea was independently invented in 1952 by Courant, Livingston and Snyder, who later acknowledged the priority of Christophilos' work.
  - Courant and Snyder wrote a follow-up paper in 1958, which contains the vast majority of the accelerator physics concepts and formalism in use to this day!
    - The LHC is perfectly understandable in terms of this paper
- Although the technique was originally formulated in terms of magnetic gradients, it's much easier to understand in terms of the separate functions of dipole and quadrupole magnets.

Combined Function vs. Separated Function

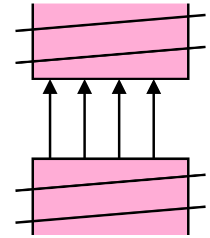
Strong focusing was originally implemented by building magnets with non-parallel pole faces to introduce a linear magnetic gradient



CERN PS (1959, 29 GeV)

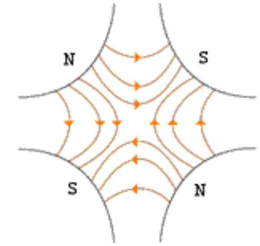
$$B_y(x) = B_0 + \frac{\partial B_y}{\partial x} x$$

=



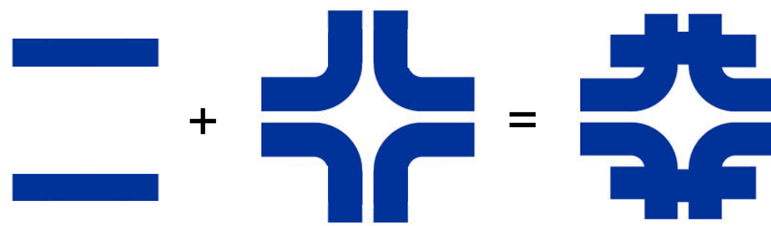
dipole

+



quadrupole

Later synchrotrons were built with physically separate dipole and quadrupole magnets. The first "separated function" synchrotron was the Fermilab Main Ring (1972, 400 GeV)

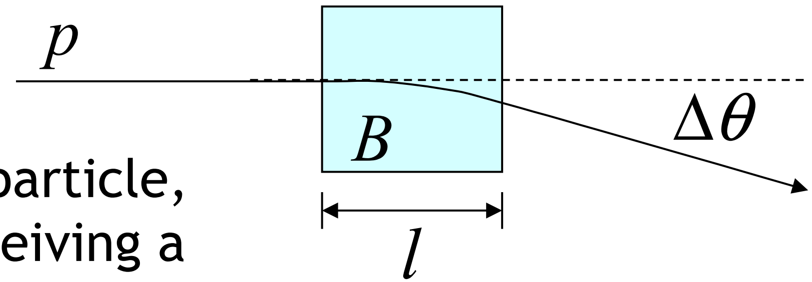


dipole quadrupole Fermilab

Strong focusing is also much easier to *teach* using separated functions, so we will…

Thin Lens Approximation and Magnetic "kick"

- If the path length through a transverse magnetic field is short compared to the bend radius of the particle, then we can think of the particle receiving a transverse "kick", which is proportional to the integrated field

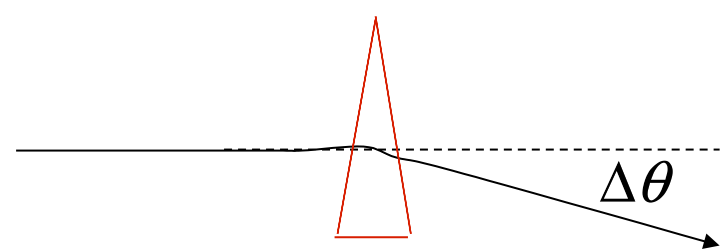


$$\boxed{p_\perp \approx qvBt = qvB(l/v) = qBl}$$

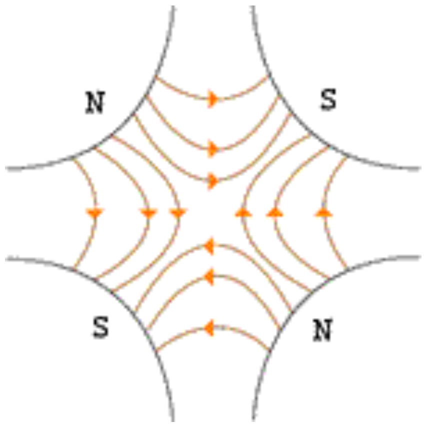
and it will be bent through small angle

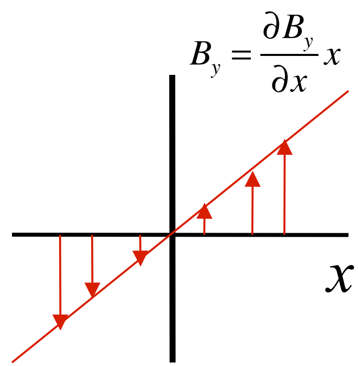
$$\boxed{\Delta\theta \approx \frac{p_\perp}{p} = \frac{Bl}{(B\rho)}}$$

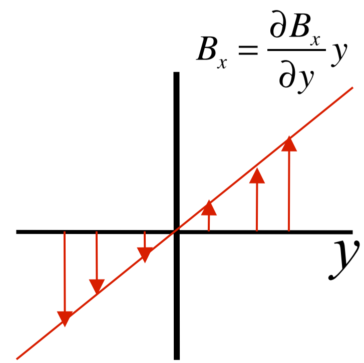
- In this "thin lens approximation", a dipole is the equivalent of a prism in classical optics.



Quadrupole Magnets* as Lenses



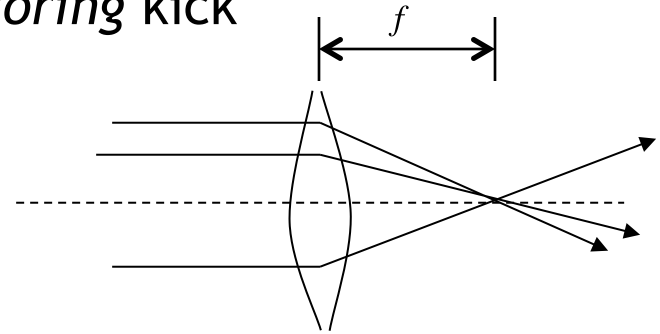




Note: $\vec{\nabla} \times \vec{B} = 0 \rightarrow \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} \equiv B'$

- A positive particle coming out of the page off center in the horizontal plane will experience a *restoring* kick *proportional to the displacement*

$$\Delta\theta \approx -\frac{B_y l}{(B\rho)} = -\frac{B' l x}{(B\rho)}$$

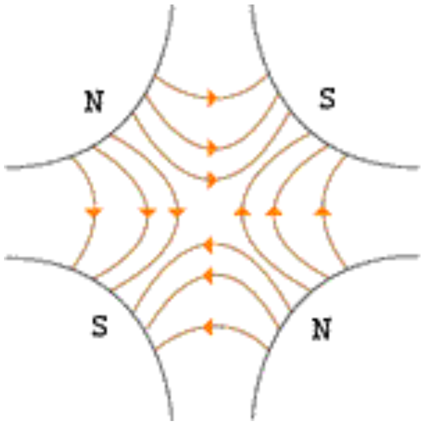


just like a "thin lens" with focal length

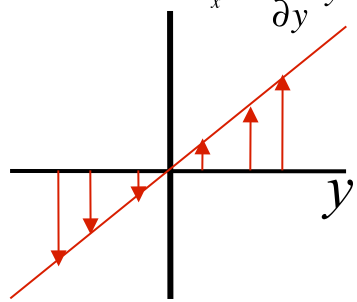
$$\boxed{f = \frac{x}{\Delta\theta} = \frac{(B\rho)}{B' l}}$$

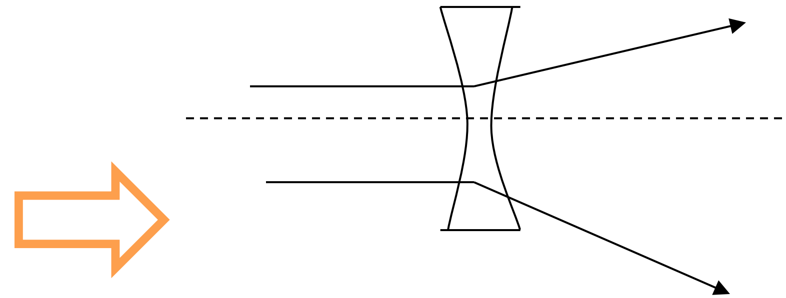
*or quadrupole term in a gradient magnet

What About the Other Plane?



$$B_x = \frac{\partial B_x}{\partial y} y$$

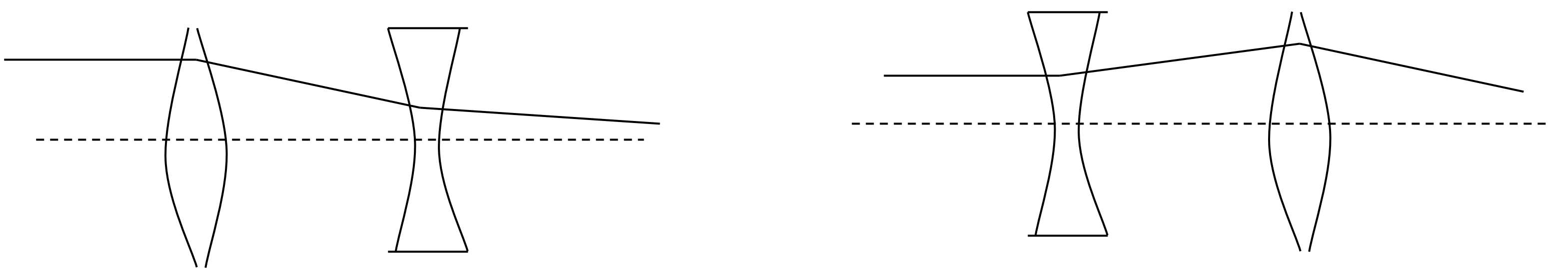




$$f = -\frac{(B\rho)}{B'l}$$

Defocusing!

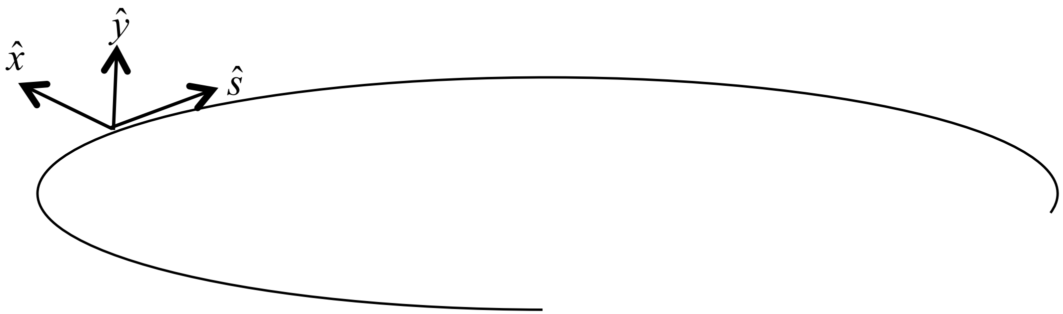
Luckily, if we place equal and opposite pairs of lenses, there will be a net focusing *regardless of the order*.



➔pairs give net focusing in *both* planes -> "FODO cell"

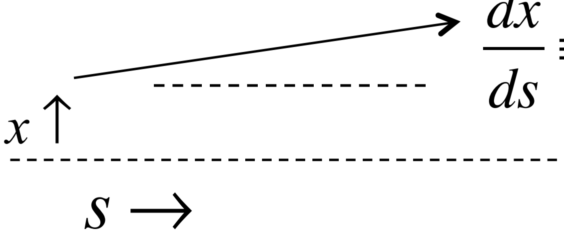
Formalism: Coordinates and Conventions

- We generally work in a right-handed coordinate system with *x* horizontal, *y* vertical, and *s* along the *nominal* trajectory.

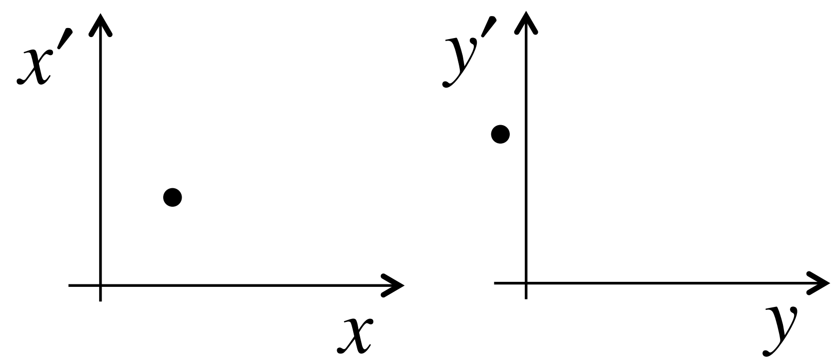


Note: *s* (rather than *t*) is the independent variable

Particle trajectory defined at any point in *s* by location in *x*,*x'* or *y*,*y'* "phase space"



$$\frac{dx}{ds} \equiv x' \approx \theta$$



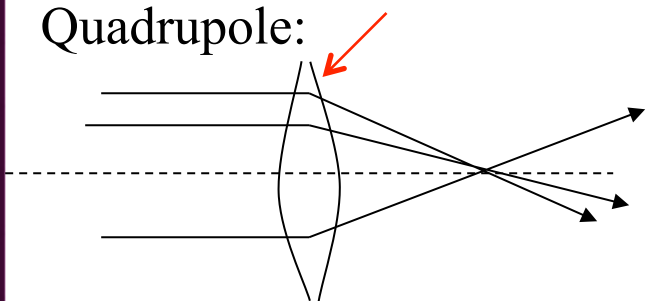
unique initial phase space point ➔ unique trajectory

Transfer Matrices

- Dipoles *define* the trajectory, so the simplest magnetic “lattice” consists of quadrupoles and the spaces in between them (drifts). We can express each of these as a linear operation in phase space.

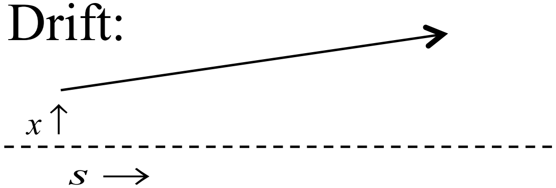
$$\Delta\theta = \Delta x' = -\frac{x}{f}$$

Quadrupole:



$$x = x(0)$$
$$x' = x'(0) - \frac{1}{f}x(0) \quad \Rightarrow \begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}\begin{pmatrix} x(0) \\ x'(0) \end{pmatrix}$$

Drift:



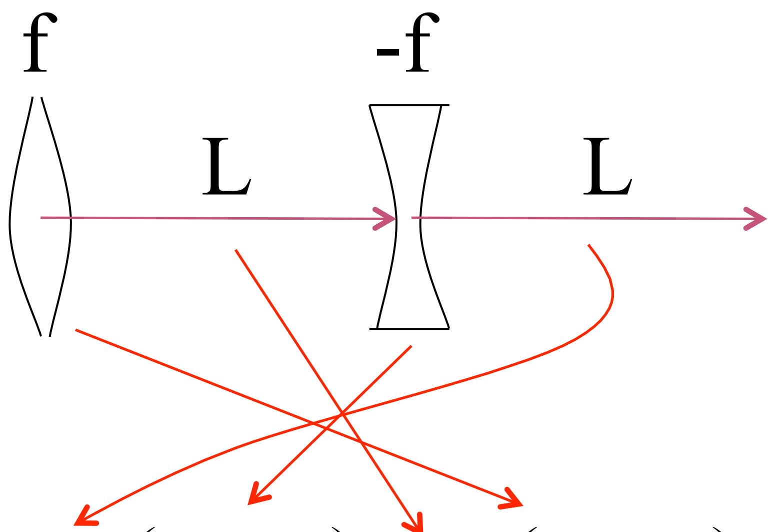
$$x(s) = x(0) + sx'(0)$$
$$x'(s) = x'(0) \quad \Rightarrow \begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x(0) \\ x'(0) \end{pmatrix}$$

- By combining these elements, we can represent an arbitrarily complex ring or line as the product of matrices.

$$\mathbf{M} = \mathbf{M}_N \ldots \mathbf{M}_2 \mathbf{M}_1$$

Example: Transfer Matrix of a FODO cell

- At the heart of every beam line or ring is the basic “FODO” cell, consisting of a focusing and a defocusing element, separated by drifts:



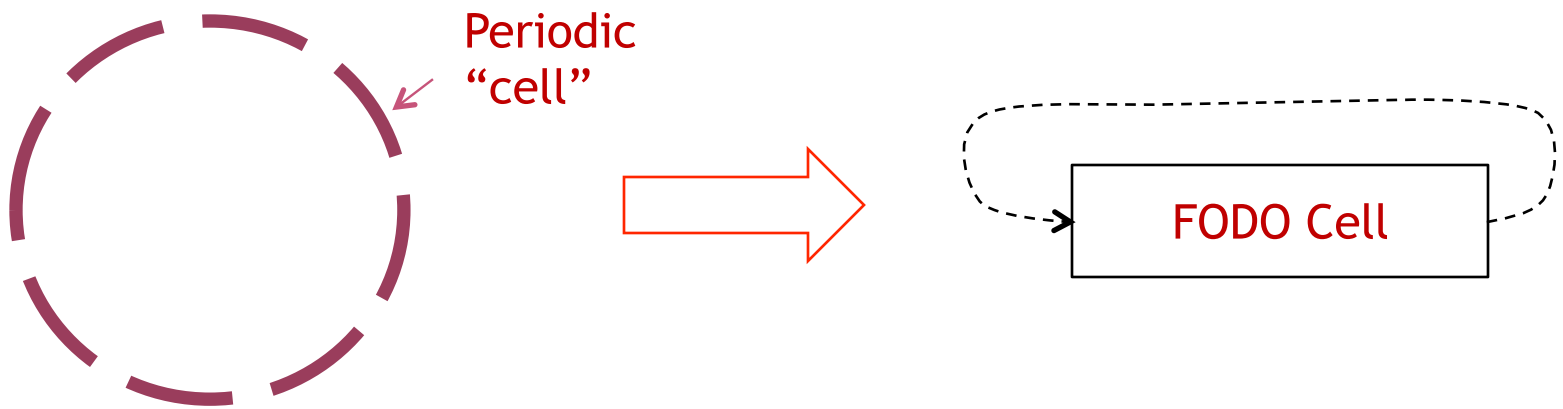
Remember: motion is usually drawn from left to right, but matrices act from right to left!

$$\Rightarrow \mathbf{M} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ +\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{L}{f} - \left(\frac{L}{f}\right)^2 & 2L + \frac{L^2}{f} \\ -\frac{L}{f^2} & 1 + \frac{L}{f} \end{pmatrix}$$

- Can build this up to describe any beam line or ring

Periodic Systems

- You might think, "Start with a beam line, then make a ring out of it."
  - Difficult to solve general case, because it depends on the initial conditions
- Therefore, we initially solve for stable motion in a *periodic* system
- We can think of a ring made of identical FODO cells as just the same cell, over and over.



$$\mathbf{M}_{ring} = \mathbf{M}_{cell}\mathbf{M}_{cell}\cdots\mathbf{M}_{cell} = \mathbf{M}_{cell}^{N}$$

- Our goal is to decouple the problem into two parts
  - The "lattice": a mathematical description of the machine itself, based only on the magnetic fields, *which is identical for each identical cell*
  - The "emittance": mathematical description for the ensemble of particles circulating in the machine.
- Extend to beam lines by using boundary conditions ("matching")

Stability Criterion

- We can represent an arbitrarily complex ring as a combination of individual matrices

$$\mathbf{M}_{ring} = \mathbf{M}_n \ldots \mathbf{M}_3 \mathbf{M}_2 \mathbf{M}_1$$

- We can express an arbitrary initial state as the sum of the eigenvectors of this matrix

$$\begin{pmatrix} x \\ x' \end{pmatrix} = A\mathbf{V_1} + B\mathbf{V_2} \Rightarrow \mathbf{M}\begin{pmatrix} x \\ x' \end{pmatrix} = A\lambda_1 \mathbf{V}_1 + B\lambda_2 \mathbf{V_2}$$

- After *n* turns, we have

$$\mathbf{M}^n \begin{pmatrix} x \\ x' \end{pmatrix} = A\lambda_1^n \mathbf{V_1} + B\lambda_2^n \mathbf{V_2}$$

- Because the individual matrices have *unit* determinants, the product must as well, so

$$\mathrm{Det}(\mathbf{M}) = \lambda_1 \lambda_2 = 1 \rightarrow \lambda_2 = 1/\lambda_1$$

Stability Criterion (cont'd)

- We can therefore express the eigenvalues as

$$\lambda_1 = e^{a}; \lambda_2 = e^{-a}; \text{ where } a \text{ is in general complex}$$

- However, if *a* has any real component, one of the solutions will grow exponentially, so the only stable values are

$$\lambda_1 = e^{i\mu}; \lambda_2 = e^{-i\mu}; \text{ where } \mu \text{ is real}$$

- Examining the (invariant) trace of the matrix

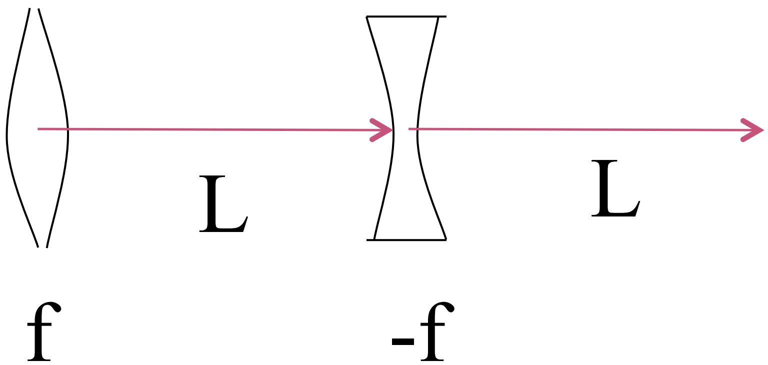
$$\mathrm{Tr}[\mathbf{M}] = e^{i\mu} + e^{-i\mu} = 2\cos\mu$$

- So the general stability criterion is simply

$$\mathrm{abs}(\mathrm{Tr}[\mathbf{M}]) < 2$$

Example

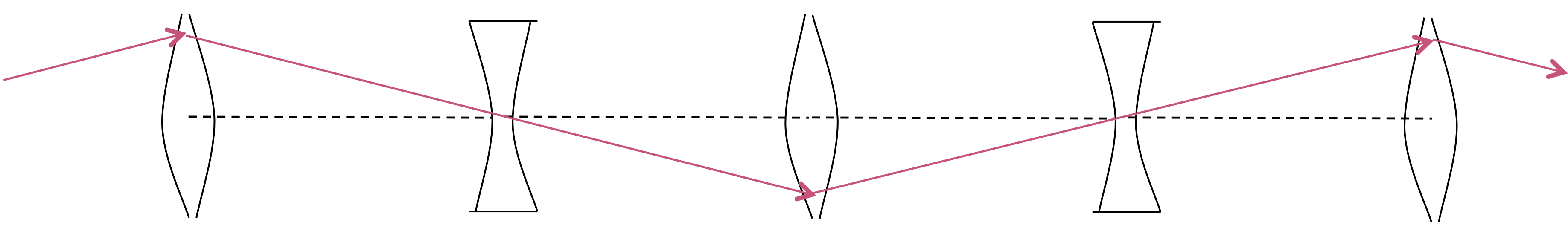
- Recall our FODO cell



$$M=\begin{pmatrix} 1-\frac{L}{f}-\left(\frac{L}{f}\right)^2 & 2L+\frac{L^2}{f} \\ -\frac{L}{f^2} & 1+\frac{L}{f} \end{pmatrix}$$

- Our stability requirement becomes

$$\mathrm{abs}\left(2-\left(\frac{L}{f}\right)^2\right)\le 2 \Rightarrow \boxed{L\le 2f}$$



Equations of Motion

- Solving for small deviations from the ideal orbit in the (curved) *x*,*y*,*s* coordinates, we get

dipole term defines radius of curvature

quadrupole term defines gradient

$$\frac{d^2x}{ds^2}+\left[\frac{1}{\rho^2}+\frac{1}{(B\rho)}\frac{\partial B_y(s)}{\partial x}\right]x=0$$

$$\frac{d^2y}{dy^2}-\frac{1}{(B\rho)}\frac{\partial B_x(s)}{\partial y}y=0$$

periodic

- This is in the form of a "Hill's Equation": $x''+K(s)x=0$
  - the most general equation for small deviations from an ideal trajectory (first used to study stability of lunar orbit)
- If *K* is constant, this is just a harmonic oscillator, and the solution is

$$x(s)=A\sin\left(\sqrt{K}\,s+\delta\right)$$

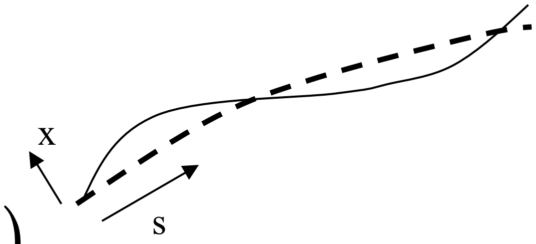
A and δ determined by initial conditions

- We therefore looks for a solution that looks "kinda sorta" like that…

General Solution: Betatron Motion

- We find (after a lot of algebra) that we can describe particle motion in terms of initial conditions and a "beta function" β(s), which is only a function of location along the nominal path.

follows periodicity of machine



Lateral deviation in one plane

$$x(s) = A\sqrt{\beta(s)}\cos\left(\psi(s)+\delta\right)$$

Phase advance

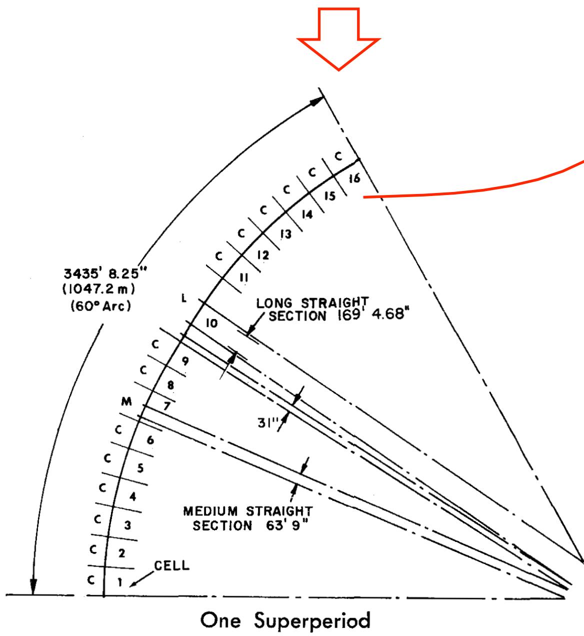
$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

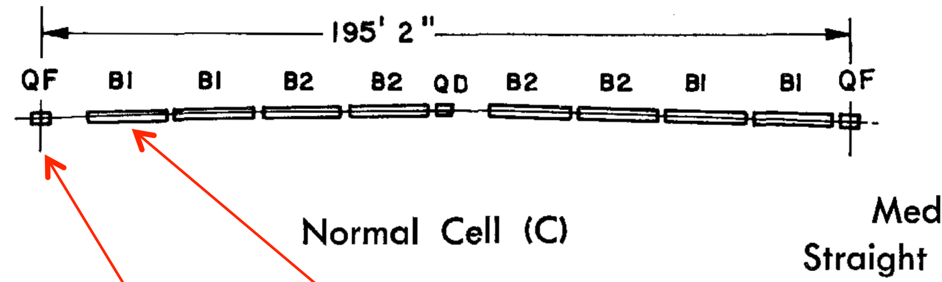
The "betatron function" $\beta(s)$ is effectively the local wavenumber and also defines the beam envelope.

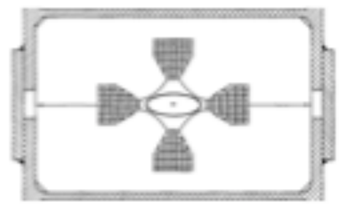
- In other words, particles undergo "pseudo-harmonic" motion about the nominal trajectory, with a variable wavelength.
- Note: β has units of [length], so the amplitude has units of [length]$^{1/2}$

Symmetric Treatment of FODO Cell

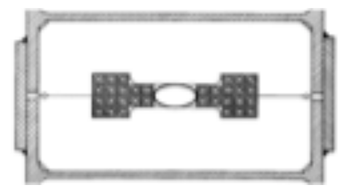
- We generally evaluate the FODO cell *center* of the focusing quad, it looks like, which makes the problem symmetric. Example: Old FNAL Main Ring



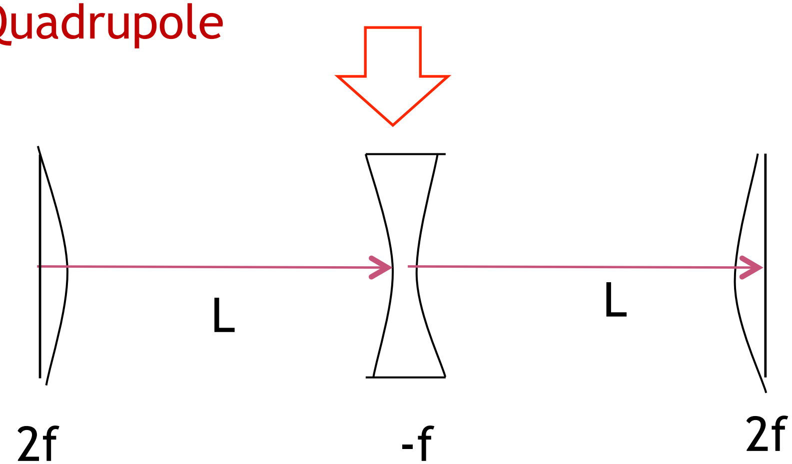




Quadrupole



Dipole



Beam Line Calculation: MAD

- We could calculate the lattice functions by hand, but…
- There have been and continue to be countless accelerator modeling programs; however MAD ("Methodical Accelerator Design"), started in 1990, continues to be the "Lingua Franca"

main_ring.madx

```
!
! One FODO cell from the FNAL Main Ring (NAL Design Report, 1968)
!
beam, particle=proton,energy=400.938272,npart=1.0E9;

LQ:=1.067;
LD:=29.74-2*LQ;

qf: QUADRUPOLE, L=LQ, K1=.0195;
d: DRIFT, L=LD;
qd: QUADRUPOLE, L=LQ, K1=-.0195;

fodo: line = (qf,d,qd,qd,d,qf);
use, period=fodo;

match,sequence=FODO;

SELECT,FLAG=SECTORMAP,clear;
SELECT,FLAG=TWISS,column=name,s,betx,alfx,bety,alfy,mux,muy;
TWISS,SAVE;

PLOT,interpolate=true,,colour=100,HAXIS=S, VAXIS1=BETX,BETY;
PLOT,interpolate=true,,colour=100,HAXIS=S, VAXIS1=ALFX,ALFY;

stop;
```

half quad
K1=1/(2f)

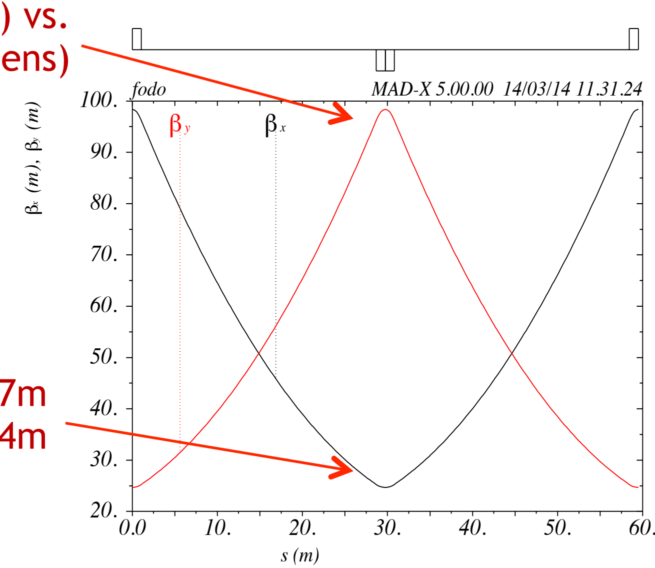
build FODO cell

force periodicity

calculate Twiss parameters

98.4m (exact) vs.
99.4m (thin lens)

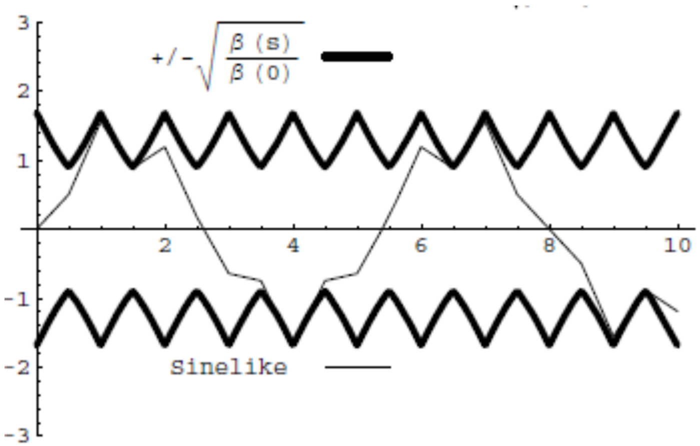
24.7m
vs. 26.4m



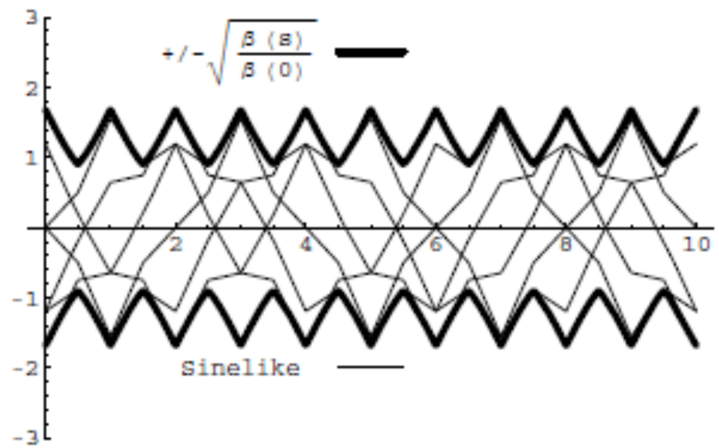
Conceptual Understanding of β

- It's important to remember that the betatron function represents a *bounding envelope* to the beam motion, not the beam motion itself

Normalized particle trajectory



Trajectories over multiple turns (or trajectories of multiple particles!)



$$x(s) = A[\beta(s)]^{1/2}\sin(\psi(s)+\delta)$$

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

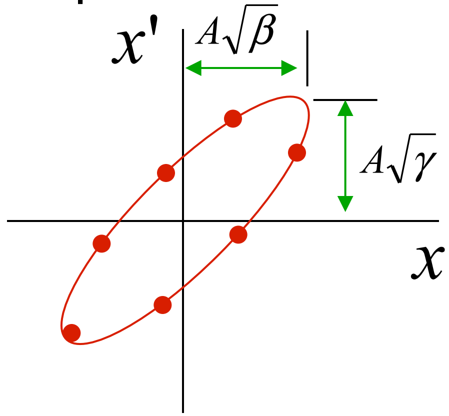
β(s) is also effectively the local wave number which determines the rate of phase advance

Closely spaced strong quads ➔ small β ➔ small aperture, lots of wiggles

Sparsely spaced weak quads ➔ large β ➔ large aperture, few wiggles

Characterizing Particle Ensembles: Emittance

- A particle returning to the same point over many terms traces an ellipse, defined by the "beta function", β, and two additional lattice parameters, α and γ.



$$\alpha = -\frac{1}{2}\frac{d\beta}{ds}$$

$$\beta x'^2 + 2\alpha x x' + \gamma x^2 = A^2 = \text{constant}$$

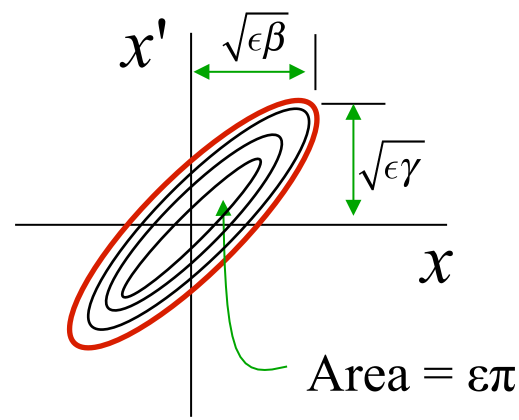
$$\gamma = \frac{1+\alpha^2}{\beta}$$

α, β, and γ = "Twiss Parameters" - NOT to be confused with relativistic β and γ!

- An ensemble of particles can characterized by a bounding ellipse, known as the "emittance"
  - Definitions vary: RMS, 95%, 99%, etc

$$\beta x' + 2\alpha x x' + \gamma x^2 = \epsilon$$

Units of length



Emittance, Beam Size, and Adiabatic Damping

- If we use the Guassian definition emittance, then the beam size is

$$\sigma_x = \sqrt{\beta_x \epsilon}$$

- Emittance is constant at a constant energy, but as particles accelerate, the emittance decreases

$$\epsilon \propto \frac{1}{\beta\gamma}$$

Relativistic β and γ
(yes, I know it's confusing)

- This is known as "adiabatic damping". We therefore define a "normalized emittance"

$$\epsilon_N \equiv \beta\gamma\epsilon$$

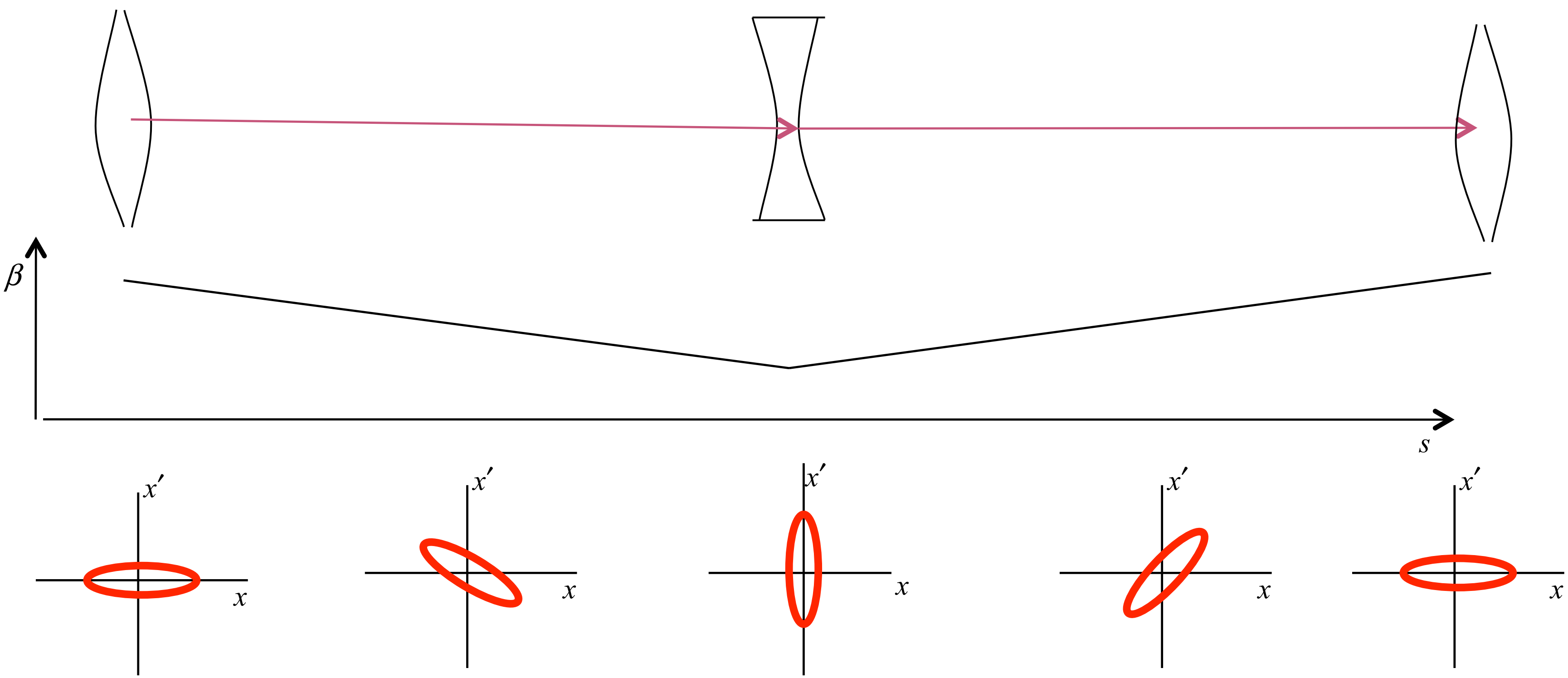
- which is constant with energy. Thus, at a particular energy

$$\sigma_x = \sqrt{\frac{\beta_x \epsilon_N}{\beta\gamma}} \propto \frac{1}{\sqrt{p}}$$

Might be a factor here; e.g. 6
for 95% emittance

Emittance and Beam Distributions

- As we go through a lattice the shape in phase space varies, by the bounding emittance remains constant



β = max
α = 0
➔maximum

β = decreasing
α >0
➔focusing

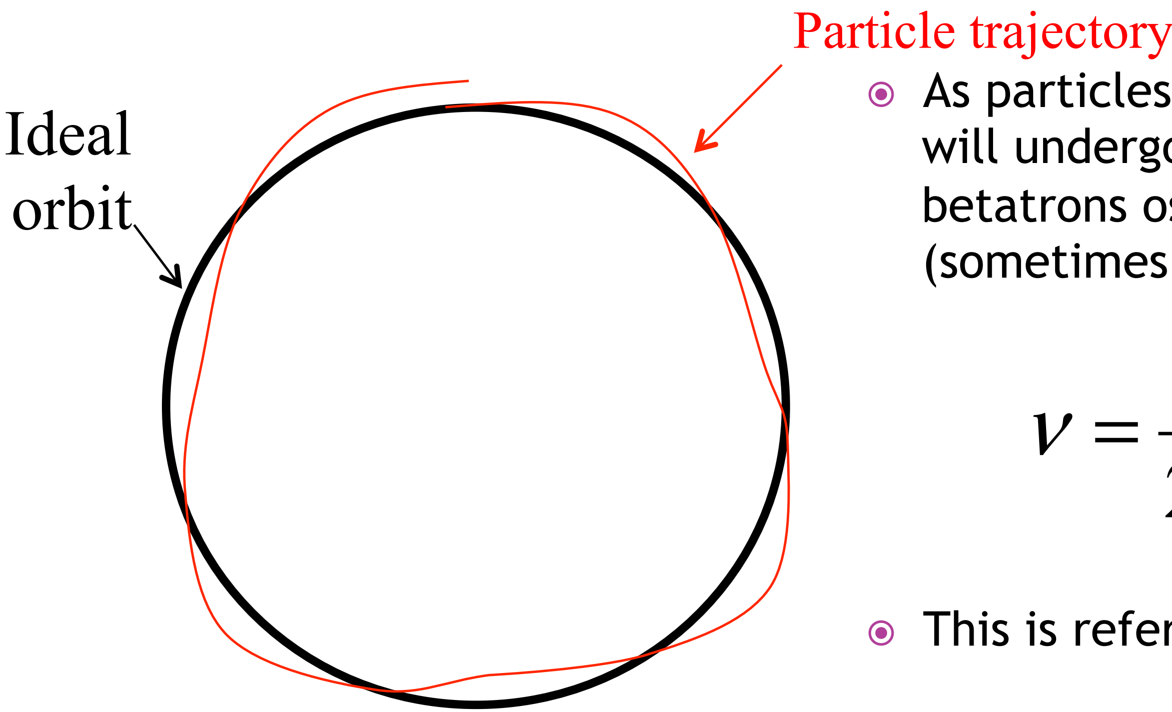
β = min
α = 0
➔minimum

β = increasing
α < 0
➔defocusing

large spatial distribution
small angular distribution

small spatial distribution
large angular distribution

Betatron Tune



- As particles go around a ring, they will undergo a number of betatrons oscillations ν (sometimes Q) given by

$$\nu = \frac{1}{2\pi}\oint \frac{ds}{\beta(s)}$$

- This is referred to as the "tune"

- We can generally think of the tune in two parts:

Integer : magnet/ aperture optimization $\nearrow$ 6.7 $\nwarrow$ Fraction: Beam Stability

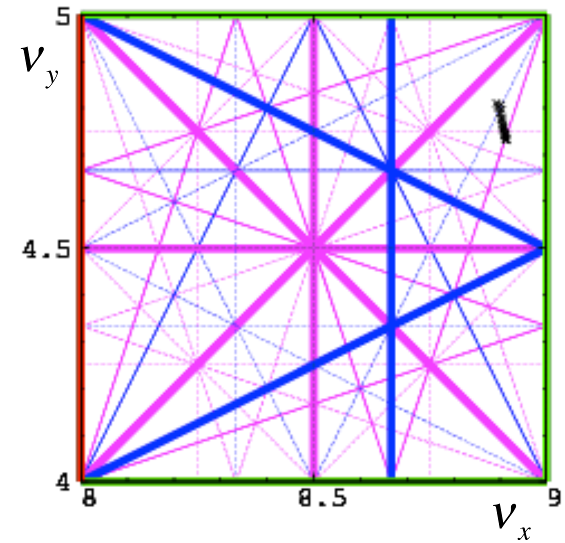
Tune, Stability, and the Tune Plane

- If the tune is an integer, or low order rational number, then the effect of any imperfection or perturbation will tend be reinforced on subsequent orbits.
- When we add the effects of coupling between the planes, we find this is also true for *combinations* of the tunes from both planes, so in general, we want to avoid

$$k_x \nu_x \pm k_y \nu_y = \text{integer} \Rightarrow \text{(resonant instability)}$$

"small" integers

➔Avoid lines in the "tune plane"



- Many instabilities occur when something perturbs the tune of the beam, or part of the beam, until it falls onto a resonance, thus you will often hear effects characterized by the "tune shift" they produce.
    - For example: the maximum tune shift sets the absolute luminosity limit in a collider (more about this in a bit…)

Off-Momentum Particles

- Our previous discussion implicitly assumed that all particles were at the same momentum
  - Each quad has a constant focal length
  - There is a single nominal trajectory
- In practice, this is never true. Particles will have a distribution about the nominal momentum, typically ~.1% or so.
- We will characterize the behavior of off-momentum particles in the following ways
  - "Dispersion" (*D*): the dependence of position on deviations from the nominal momentum

$$\Delta x(s) = D_x(s)\frac{\Delta p}{p_0}$$

Dispersion has units of length

D has units of length

  - "Chromaticity" (η) : the change in the tune caused by the different focal lengths for off-momentum particles (the focal length goes up with momentum)

$$\Delta \upsilon_x = \xi_x \frac{\Delta p}{p_0} \quad \left(\text{sometimes} \frac{\Delta \upsilon_x}{\upsilon_x} = \xi_x \frac{\Delta p}{p_0} \right)$$

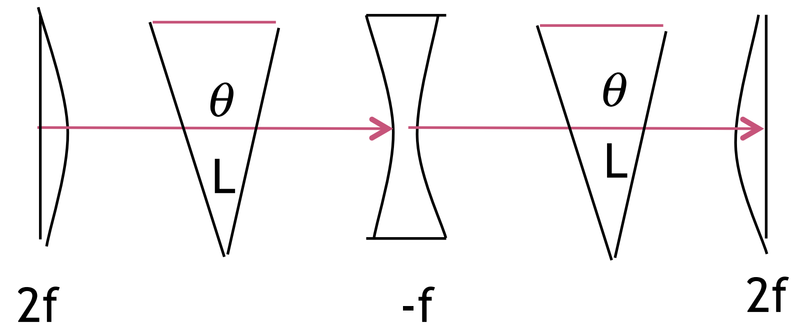
  - Path length changes ("momentum compaction")

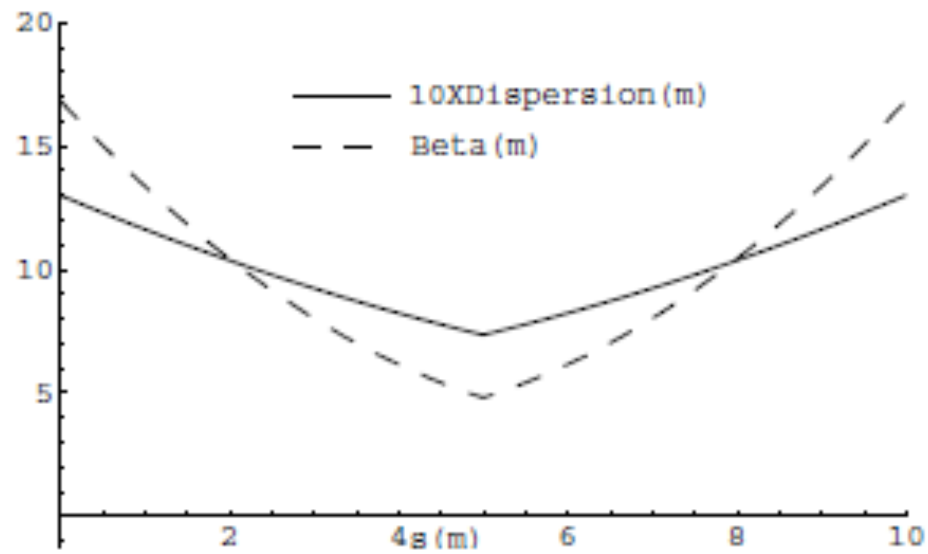
$$\frac{\Delta L}{L} = \alpha_C \frac{\Delta p}{p}$$

We overloaded β and γ; might as well overload α, too.

Off-Momentum Particles (cont'd)

- The chromaticity (ξ) and the momentum compaction (a) are properties of the entire ring. However, the dispersion ($D(s)$) is another position dependent lattice function, which follows the periodicity of the machine.
- If we look at our standard FODO cell, but include the bend magnets, we find that the dispersion functions ~track the beta functions

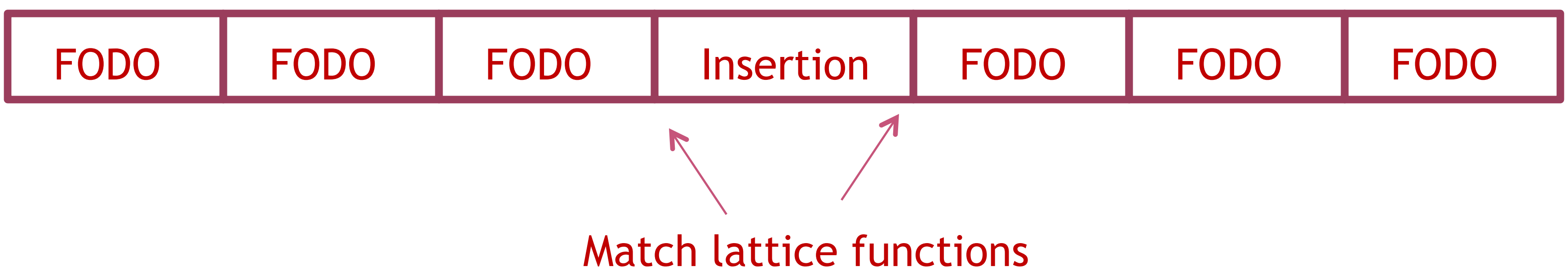




- Typically dispersion is ~meters and momentum spread is ~.1%, so motion due to dispersion is ~mm

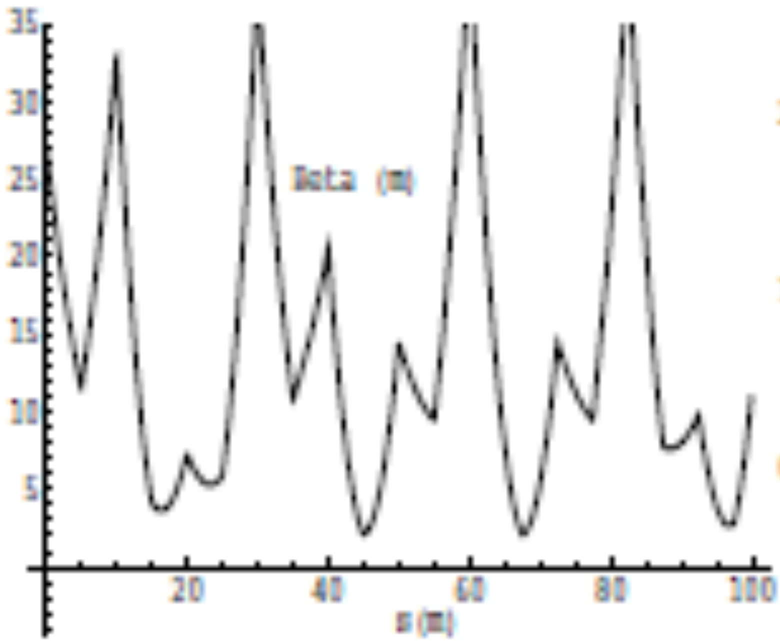
Insertions

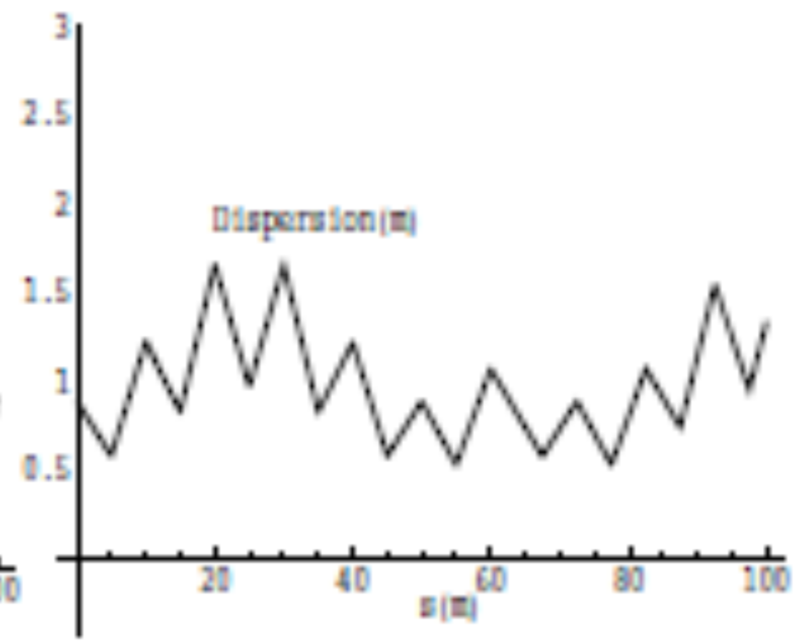
- So far, we've talked about nice, periodic lattice, but that may not be all that useful in the real world. In particular, we generally want
  - Locations for injection of extraction.
  - "Straight" sections for RF, instrumentation, etc
  - Low beta points for collisions
- Since we generally think of these as taking the place of things in our lattice, we call them "insertions"



Mismatch and Beta Beating

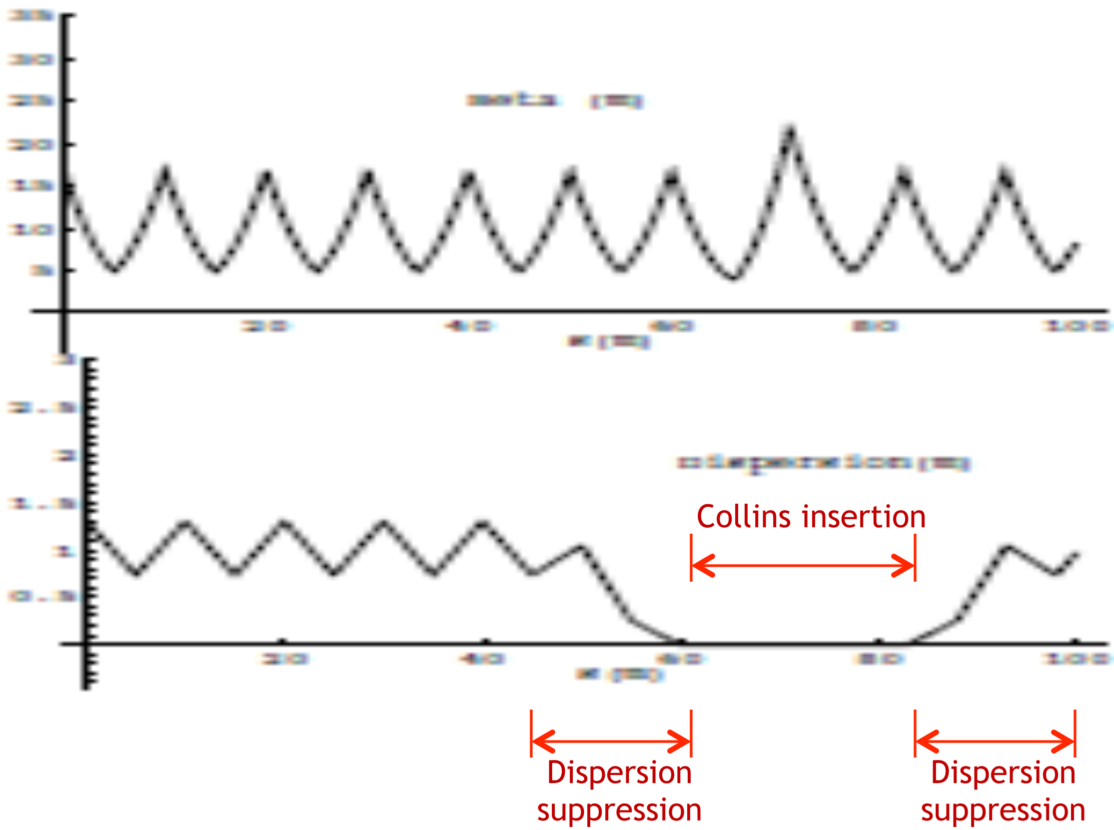
- Simply modifying a section of the lattice without matching will result in a distortion of the lattice functions around the ring (sometimes called “beta beating”)
- Here’s an example of increasing the drift space in one FODO cell from 5 to 7.5 m





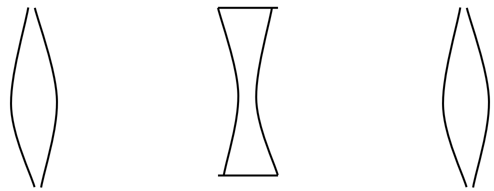
Straight Section: Collins Insertion

- A Collins insertion is an arrangement magnets designed to insert a straight section, while matching lattice parameters. It's also necessary to modify the dipoles on either side of the straight section to "suppress" dispersion.



Focusing Triplet

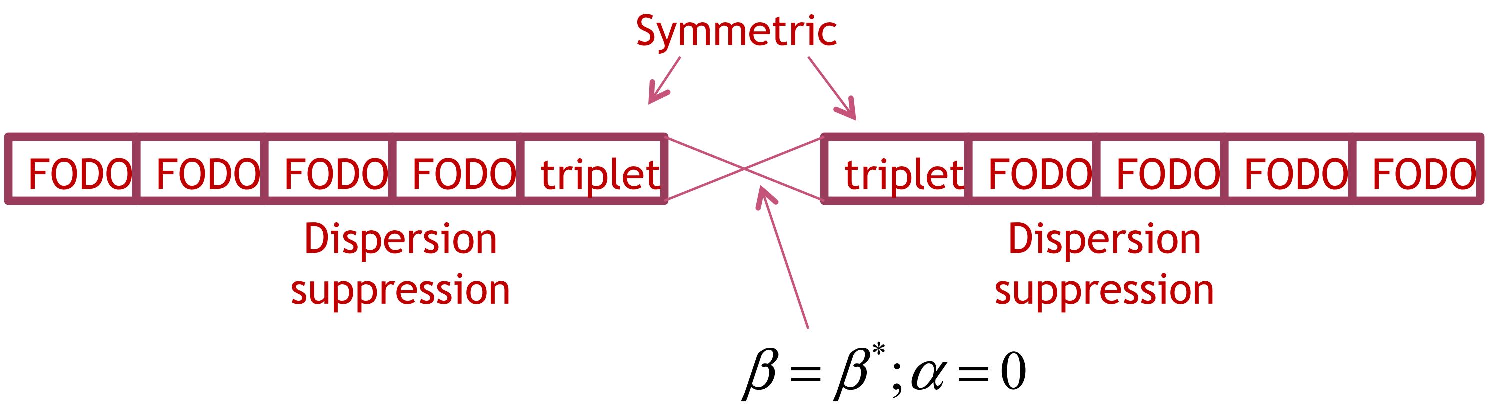
- In experimental applications, we will often want to focus beam down to a waist (minimum β) in both planes. In general, we can accomplish this with a triplet of quadrupoles.



- Such triplets are a workhorse in beam lines, and you'll see them wherever you want to focus beam down to a point.
  - Can also be used to match lattice functions between dissimilar beam line segments
- The solution, starting with a arbitrary lattice functions, is not trivial and in general these problems are solved numerically (eg, MAD can do this)

Low β Insertions

- In a collider, we will want to focus the beam in both planes as small as possible.
- This can be done with a symmetric pair of focusing triplets, matched to the lattice functions (dispersion suppression is assumed)



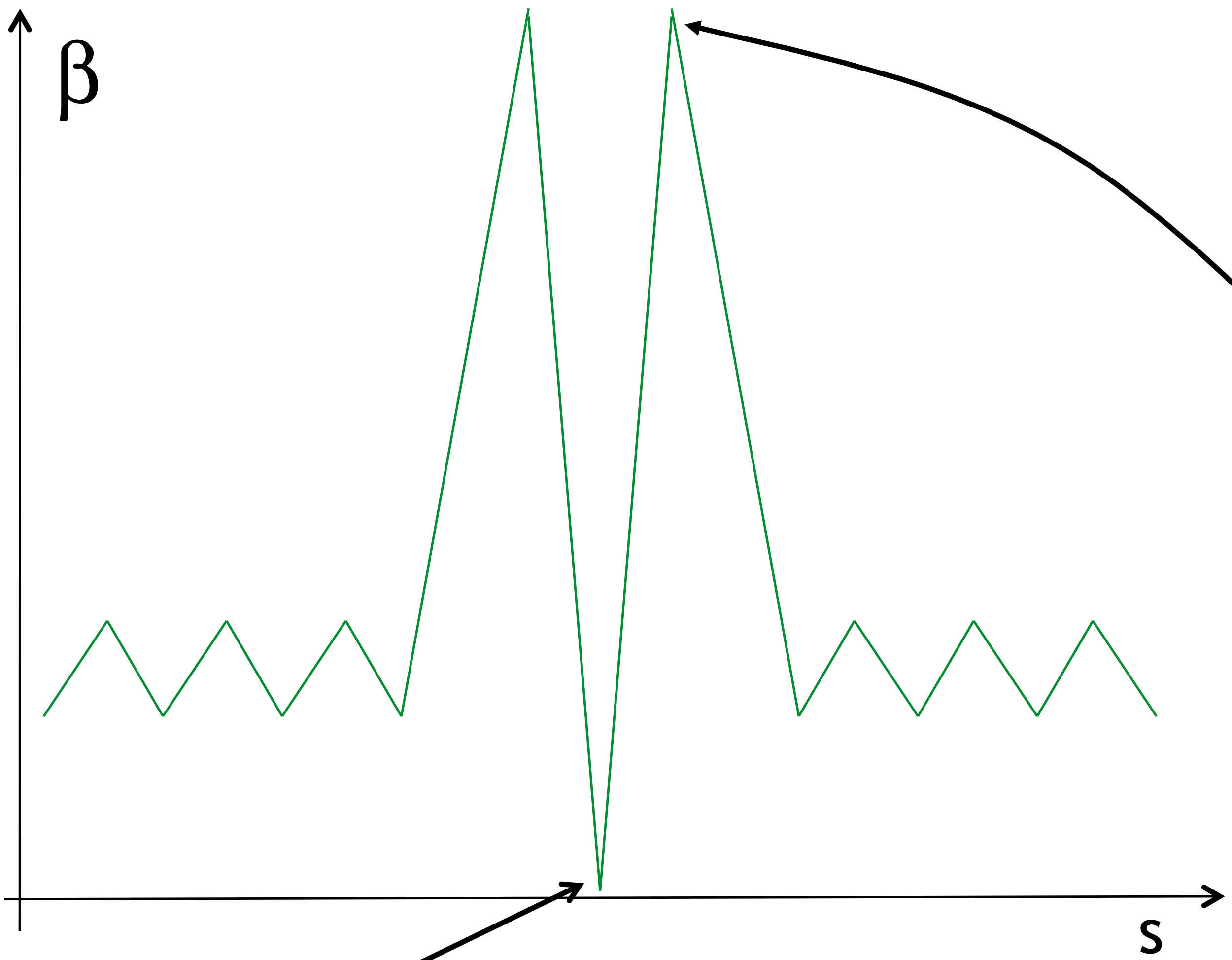
- Near the focus, β evolves as

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

where *s* is measured from the location of the waist

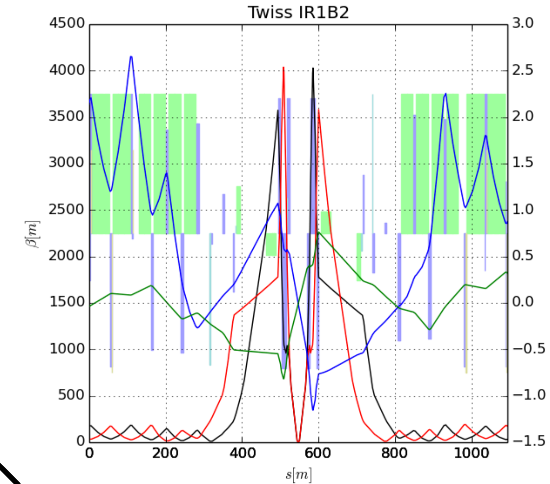
- This means that the smaller I make β*, *the bigger the beam gets in the focusing triplets!*
  - We'll discuss this much more shortly

Behavior near low-β insertion



$$\beta(\Delta s) = \beta^* + \frac{\Delta s^2}{\beta^*}$$

➔ small β* means large β (aperture) at focusing triplet



LHC

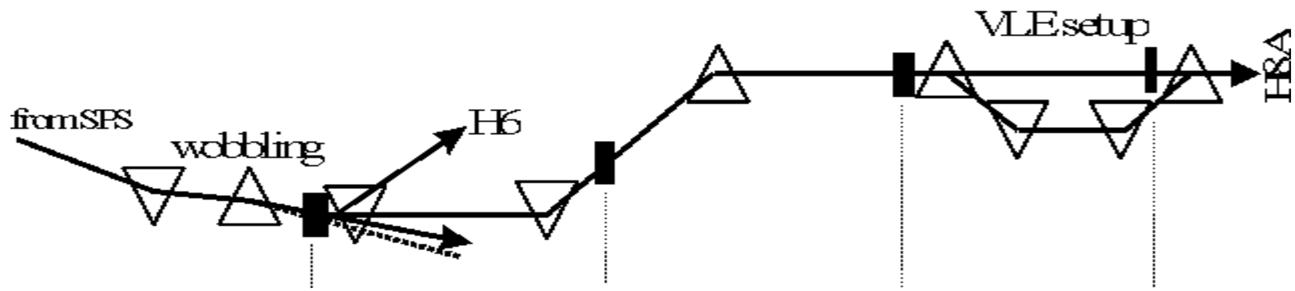
Much more about this shortly!

Beam Lines

- In our definition and derivation of the lattice function, a closed path through a periodic system. This definition doesn't exist for a beam line, but once we know the lattice functions at one point, we know how to propagate the lattice function down the beam line.

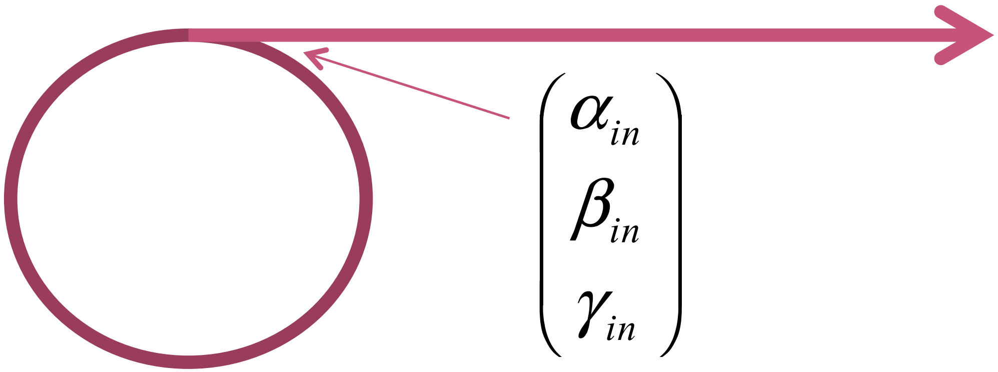
$$\mathbf{M}(out, in)$$

$$\begin{pmatrix} \alpha_{in} \\ \beta_{in} \\ \gamma_{in} \end{pmatrix} \qquad \begin{pmatrix} \alpha_{out} \\ \beta_{out} \\ \gamma_{out} \end{pmatrix}$$



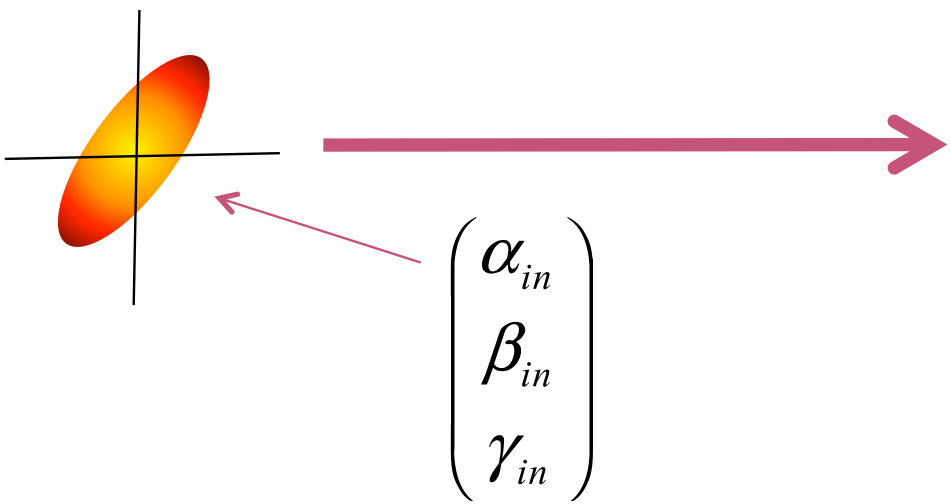
Establishing Initial Conditions

- When extracting beam from a ring, the initial optics of the beam line are set by the optics at the point of extraction.



$$\begin{pmatrix} \alpha_{in} \\ \beta_{in} \\ \gamma_{in} \end{pmatrix}$$

- For particles from a source, the initial lattice functions can be defined by the distribution of the particles out of the source



$$\begin{pmatrix} \alpha_{in} \\ \beta_{in} \\ \gamma_{in} \end{pmatrix}$$