Hadron Colliders

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LHC Interaction Region

Relative beam sizes around IP1 (Atlas) in collision
These lectures will focus on the underlying physics and evolution of the highest energy hadron accelerators

- This has largely driven the development of the technology; however
- High energy research machines are a tiny fraction (~1%) of the particle accelerators in use today.

I’ll be fairly rigorous

- In the end, you should have a reasonably quantitative understanding of most of the accelerator jargon you’ll hear in a typical high energy physics talk:
  - “Lattice”
  - “Beta function”
  - “Tune” and “Tune shift”
  - “Emittance”
  - “RF”
  - “Luminosity”
  - “Squeeze”
  - etc.
Outline of Lectures

- Accelerator physics basics
  - Transverse motion
  - Longitudinal motion
  - Colliding beams

- LHC specific topics
  - Maximizing luminosity
  - Upgrade plans

- Special topics
  - Tricks of the trade
  - Instrumentation
  - etc
Some Perspective: Just the Tip of the Iceberg

Number of accelerators worldwide
~ 26,000

- 41%
- 44%
- 9%
- 1%
- 4%

Annual growth is several percent
Sales >3.5 B$/yr
Value of treated good > 50 B$/yr **

- Radiotherapy (>100,000 treatments/yr)*
- Medical Radioisotopes
- Research (incl. biomedical)
- >1 GeV for research
- Industrial Processing and Research
- Ion Implanters & Surface Modification
Going to higher energies = going back in time
Accelerators allow us to go back 13.8 billion years and recreate conditions that existed a few trillionths of a second after the Big Bang — the place where our current understanding of physics breaks down.

In addition to high energy, we need high “luminosity” that is, lots of particles interacting, to see rare processes.
Basic Relativity

\[ \beta \equiv \frac{\nu}{c} \]

\[ \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}} \]

- Momentum: \( p = \gamma m v \)
- Total energy: \( E = \gamma m c^2 \)
- Kinetic energy: \( K = E - m c^2 \)

\[ E^2 = \sqrt{(mc^2)^2 + (pc)^2} \]

Units

- For the most part, we will use SI units, except:
  - Energy: eV (keV, MeV, etc) [1 eV = 1.6x10^{-19} J]
  - Mass: eV/c^2 [proton = 1.67x10^{-27} kg = 938 MeV/c^2]
  - Momentum: eV/c [proton @ \( \beta = .9 \) = 1.94 GeV/c]

- In the US and Europe, we normally talk about the kinetic energy (K) of a particle beam, although we’ll see that momentum really makes more sense.

Some Handy Relationships

\[ \beta = \frac{pc}{E} \]

\[ \gamma = \frac{E}{mc^2} \]

\[ \beta \gamma = \frac{pc}{mc^2} \]

These units make these relationships really easy to calculate.
Built at CERN, straddling the French/Swiss border

- 27 km in circumference
- Has collided two proton beams at 4000 GeV each
- In 2015, will reach (almost) design energy of 7000 GeV/beam.
- That’s where we are. Now let’s see how we got here...
The first artificial acceleration of particles was done using “Crookes tubes”, in the latter half of the 19th century:
- These were used to produce the first X-rays (1875)
- At the time no one understood what was going on

The first “particle physics experiment” told Ernest Rutherford the structure of the atom (1911)

In this case, the “accelerator” was a naturally decaying $^{235}\text{U}$ nucleus
Radioactive sources produce maximum energies of a few million electron volts (MeV).

Cosmic rays reach energies of ~1,000,000,000 x LHC but the rates are too low to be useful as a study tool.

- Remember what I said about “luminosity”.

However, low energy cosmic rays are extremely useful for detector testing, commissioning, etc.
The simplest accelerators accelerate charged particles through a static electric field. Example: vacuum tubes (or CRT TV’s)

Limited by magnitude of electric field:
- CRT display ~keV
- X-ray tube ~10’s of keV
- Van de Graaf ~MeVs

Solutions:
- Alternate fields to keep particles in accelerating fields -> Radio Frequency (RF) acceleration
- Bend particles so they see the same accelerating field over and over -> cyclotrons, synchrotrons
Interlude: Electrons vs. Protons

- Electrons are point-like
  - Well-defined initial state
  - Full energy available to interaction

- Protons are made of quarks and gluons
  - Interaction take place between these constituents.
  - Only a small fraction of energy available, not well-defined.
  - Rest of particle fragments -> big mess!

So why not stick to electrons?
As the trajectory of a charged particle is deflected, it emits “synchrotron radiation”

\[
\text{Radiated Power} \propto \frac{1}{\rho^2} \left(\frac{E}{m}\right)^4
\]

An electron will radiate about \(10^{13}\) times more power than a proton of the same energy!!!!

- **Protons:** Synchrotron radiation does not affect kinematics very much
  - Energy limited by strength of magnetic fields and size of ring

- **Electrons:** Synchrotron radiation dominates kinematics
  - To go higher energy, we have to *lower* the magnetic field and go to *huge* rings
  - Eventually, we lose the benefit of a circular accelerator, because we lose all the energy each time around.

Since the beginning, the “energy frontier” has belonged to proton (and/or antiproton) machines, while electrons are used for precision studies and other purposes.

Now, back to the program...
The Cyclotron (1930’s)

A charged particle in a uniform magnetic field will follow a circular path of radius

\[ \rho = \frac{p}{qB} \approx \frac{mv}{qB} \quad (v \ll c) \]

\[ f = \frac{v}{2\pi \rho} = \frac{qB}{2\pi m} \quad \text{(constant!!)} \]

\[ \Omega_s = \frac{f}{2\pi} = \frac{qB}{m} \]

For a proton:

\[ f_c = 15.2 \times B[T] \quad \text{MHz} \]

i.e. “RF” range

would not work for electrons!

“Cyclotron Frequency”

Accelerating “DEES”: by applying a voltage which oscillates at \( f_c \), we can accelerate the particle a little bit each time around, allowing us to get to high energies with a relatively small voltage.

HCPSS, August 11-22, 2014  E. Prebys, Hadron Colliders, Lecture 1
Round and Round We Go: the First Cyclotrons

- ~1930 (Berkeley)
  - Lawrence and Livingston
  - K=80KeV
  - Fit in your hand

- 1935 - 60” Cyclotron
  - Lawrence, et al. (LBL)
  - ~19 MeV (D.)
  - Prototype for many
Cyclotrons were limited by three problems

- Constant frequency breaks down at ~10% speed of light
  - Solved with variable frequency “synchro-cyclotrons”
    - phase stability (more about this later)
- As energy goes up, magnet gets huge
- Beams are not well focused and get larger with energy

Two major advances allowed accelerators to go beyond the energies and intensities possible at cyclotrons

- “Synchrotron” - in which the magnetic field is increased as the energy increases (proportional to momentum), such that particles continue to follow the same path. (McMillan, 1945)
- “Strong focusing” - a technique in which magnetic gradients (non-uniform fields) are used to focus particles and keep them in a smaller beam pipe than was possible with cyclotrons. (Christofilos, 1949; Courant, Livingston, and Snyder, 1952)

Note: still plenty of uses for cyclotrons (simple, inexpensive, rapid cycling)

- Medical treatments
- Isotope production
- Nuclear physics

Onward and Upward!
The relativistically correct form of Newton’s Laws for a particle in an electromagnetic field is:

\[ \vec{F} = \frac{d\vec{p}}{dt} = q \left( \vec{E} + \vec{v} \times \vec{B} \right); \quad \vec{p} = \gamma m\vec{v} \]

A particle of unit charge in a uniform magnetic field will move in a circle of radius

\[ \rho = \frac{p}{eB} \]

\[ (B\rho) = \frac{p}{e} \]

constant for fixed energy!

T-m\(^2\)/s=V

\[ (B\rho)_c = \frac{pc}{e} \]

units of eV in our usual convention

Beam “rigidity” = constant at a given momentum (even when \(B=0\)!

If all magnetic fields are scaled with the momentum as particles accelerate, the trajectories remain the same ➔ “synchrotron”
Example Beam Parameters

- Compare Fermilab LINAC (K=400 MeV) to LHC (K=7000 GeV)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Equation</th>
<th>Injection</th>
<th>Extraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>proton mass</td>
<td>m</td>
<td>( m )</td>
<td>0.938</td>
<td></td>
</tr>
<tr>
<td>kinetic energy</td>
<td>( K )</td>
<td>( K + mc^2 )</td>
<td>.4</td>
<td>7000</td>
</tr>
<tr>
<td>total energy</td>
<td>( E )</td>
<td>( E^2 - mc^2 )</td>
<td>1.3382</td>
<td>7000.938</td>
</tr>
<tr>
<td>momentum</td>
<td>( p )</td>
<td>( \frac{pc}{E} )</td>
<td>0.95426</td>
<td>7000.938</td>
</tr>
<tr>
<td>rel. beta</td>
<td>( \beta )</td>
<td>( \frac{pc}{E} )</td>
<td>0.713</td>
<td>0.9999999991</td>
</tr>
<tr>
<td>rel. gamma</td>
<td>( \gamma )</td>
<td>( \frac{E}{mc^2} )</td>
<td>1.426</td>
<td>7461.5</td>
</tr>
<tr>
<td>beta-gamma</td>
<td>( \beta \gamma )</td>
<td>( \frac{pc}{mc^2} )</td>
<td>1.017</td>
<td>7461.5</td>
</tr>
<tr>
<td>rigidity</td>
<td>((B\rho) )</td>
<td>( \frac{p[GeV]}{0.2997} )</td>
<td>3.18</td>
<td>23353.</td>
</tr>
</tbody>
</table>

This would be the radius of curvature in a 1 T magnetic field or the field in Tesla needed to give a 1 m radius of curvature.
Weak Focusing

- Cyclotrons relied on the fact that magnetic fields between two pole faces are never perfectly uniform.
- This prevents the particles from spiraling out of the pole gap.
- In early synchrotrons, radial field profiles were optimized to take advantage of this effect, but in any weak focused beams, the beam size grows with energy.
- The most famous weak focusing synchrotron was the Berkeley Bevatron, which had a kinetic energy of 6.2 GeV
  - High enough to make antiprotons (and win a Nobel Prize)
  - It had an aperture 12”x48”!
**Strong Focusing**

- Strong focusing utilizes alternating magnetic gradients to precisely control the focusing of a beam of particles
  - The principle was first developed in 1949 by Nicholas Christofilos, a Greek-American engineer, who was working for an elevator company in Athens at the time.
  - Rather than publish the idea, he applied for a patent, and it went largely ignored.
  - The idea was independently invented in 1952 by Courant, Livingston and Snyder, who later acknowledged the priority of Christofilos’ work.
  - Courant and Snyder wrote a follow-up paper in 1958, which contains the vast majority of the accelerator physics concepts and formalism in use to this day!
    - The LHC is perfectly understandable in terms of this paper
- Although the technique was originally formulated in terms of magnetic gradients, it’s much easier to understand in terms of the separate functions of dipole and quadrupole magnets.
Combined Function vs. Separated Function

Strong focusing was originally implemented by building magnets with non-parallel pole faces to introduce a linear magnetic gradient

\[ B_y(x) = B_0 + \frac{\partial B_y}{\partial x} x \]

Later synchrotrons were built with physically separate dipole and quadrupole magnets. The first “separated function” synchrotron was the Fermilab Main Ring (1972, 400 GeV)

Strong focusing is also much easier to teach using separated functions, so we will...
If the path length through a transverse magnetic field is short compared to the bend radius of the particle, then we can think of the particle receiving a transverse “kick”, which is proportional to the integrated field.

\[ p_\perp \approx qvBt = qvB(l/v) = qBl \]

and it will be bent through small angle \( \Delta \theta \approx \frac{p_\perp}{p} = \frac{Bl}{(B\rho)} \)

In this “thin lens approximation”, a dipole is the equivalent of a prism in classical optics.
A positive particle coming out of the page off center in the horizontal plane will experience a *restoring* kick proportional to the displacement

\[ \Delta \theta \approx -\frac{B_y l}{(B \rho)} = -\frac{B' l x}{(B \rho)} \]

just like a “thin lens” with focal length

\[ f = \frac{x}{\Delta \theta} = \frac{(B \rho)}{B' l} \]

*or quadrupole term in a gradient magnet*
Luckily, if we place equal and opposite pairs of lenses, there will be a net focusing *regardless of the order*.

\[ B_x = \frac{\partial B_x}{\partial y} \]

Pairs give net focusing in *both* planes -> “FODO cell”
We generally work in a right-handed coordinate system with $x$ horizontal, $y$ vertical, and $s$ along the *nominal* trajectory.

![Diagram of particle trajectory](image)

Note: $s$ (rather than $t$) is the independent variable.

Particle trajectory defined at any point in $s$ by location in $x,x'$ or $y,y'$ “phase space”

unique initial phase space point $\Rightarrow$ unique trajectory
Transfer Matrices

- Dipoles define the trajectory, so the simplest magnetic “lattice” consists of quadrupoles and the spaces in between them (drifts). We can express each of these as a linear operation in phase space.

\[ \Delta \theta = \Delta x' = -\frac{x}{f} \]

**Quadrupole:**
\[ x = x(0) \]
\[ x'(s) = x'(0) - \frac{1}{f} x(0) \]
\[ \Rightarrow \begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x(0) \\ x'(0) \end{pmatrix} \]

**Drift:**
\[ x(s) = x(0) + sx'(0) \]
\[ x'(s) = x'(0) \]
\[ \Rightarrow \begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x(0) \\ x'(0) \end{pmatrix} \]

- By combining these elements, we can represent an arbitrarily complex ring or line as the product of matrices.

\[ M = M_N \ldots M_2 M_1 \]
At the heart of every beam line or ring is the basic “FODO” cell, consisting of a focusing and a defocusing element, separated by drifts:

\[
\begin{pmatrix}
  f \\
  \hline
  \phantom{f} & L & \phantom{f}
\end{pmatrix}
\begin{pmatrix}
  -f \\
  \hline
  \phantom{-f} & L & \phantom{-f}
\end{pmatrix}
\]

\[
M = \begin{pmatrix}
  1 & L \\
  0 & 1
\end{pmatrix}
+ \frac{1}{f}
\begin{pmatrix}
  1 & 0 \\
  0 & 1
\end{pmatrix}
\begin{pmatrix}
  1 & L \\
  0 & 1
\end{pmatrix}
- \frac{1}{f}
\begin{pmatrix}
  1 & 0 \\
  0 & 1
\end{pmatrix}

= \begin{pmatrix}
  1-\frac{L}{f}-\left(\frac{L}{f}\right)^2 & 2L+\frac{L^2}{f} \\
  \frac{L}{f^2} & 1+\frac{L}{f}
\end{pmatrix}
\]

Remember: motion is usually drawn from left to right, but matrices act from right to left!

Can build this up to describe any beam line or ring.
Periodic Systems

- You might think, “Start with a beam line, then make a ring out of it.”
  - Difficult to solve general case, because it depends on the initial conditions
- Therefore, we initially solve for stable motion in a **periodic** system
- We can think of a ring made of identical FODO cells as just the same cell, over and over.

\[
M_{\text{ring}} = M_{\text{cell}}M_{\text{cell}} \cdots M_{\text{cell}} = M_{\text{cell}}^N
\]

- Our goal is to decouple the problem into two parts
  - The “**lattice**”: a mathematical description of the machine itself, based only on the magnetic fields, *which is identical for each identical cell*
  - The “**emittance**”: mathematical description for the ensemble of particles circulating in the machine.
- Extend to beam lines by using boundary conditions ("matching")
Stability Criterion

- We can represent an arbitrarily complex ring as a combination of individual matrices

\[
M_{ring} = M_n \ldots M_3 M_2 M_1
\]

- We can express an arbitrary initial state as the sum of the eigenvectors of this matrix

\[
\begin{pmatrix}
    x \\
    x'
\end{pmatrix}
= AV_1 + BV_2 \implies M
\begin{pmatrix}
    x \\
    x'
\end{pmatrix}
= A\lambda_1 V_1 + B\lambda_2 V_2
\]

- After \( n \) turns, we have

\[
M^n
\begin{pmatrix}
    x \\
    x'
\end{pmatrix}
= A\lambda_1^n V_1 + B\lambda_2^n V_2
\]

- Because the individual matrices have *unit* determinants, the product must as well, so

\[
\text{Det}(M) = \lambda_1 \lambda_2 = 1 \rightarrow \lambda_2 = 1 / \lambda_1
\]
Stability Criterion (cont’d)

- We can therefore express the eigenvalues as
  \[ \lambda_1 = e^a; \lambda_2 = e^{-a}; \]  
  where \( a \) is in general complex

- However, if \( a \) has any real component, one of the solutions will grow exponentially, so the only stable values are
  \[ \lambda_1 = e^{i\mu}; \lambda_2 = e^{-i\mu}; \]  
  where \( \mu \) is real

- Examining the (invariant) trace of the matrix
  \[ \text{Tr}[M] = e^{i\mu} + e^{-i\mu} = 2 \cos \mu \]

- So the general stability criterion is simply
  \[ \text{abs(Tr}[M]) < 2 \]
Recall our FODO cell

\[
\begin{pmatrix}
\begin{array}{c}
\text{f} \\
L \\
-\text{f}
\end{array}
\end{pmatrix}
\]

Our stability requirement becomes

\[
\begin{align*}
\left| 2 - \left( \frac{L}{f} \right)^2 \right| & \leq 2 \Rightarrow \boxed{L \leq 2f}
\end{align*}
\]
Solving for small deviations from the ideal orbit in the \((x,y,s)\) coordinates, we get

\[
\frac{d^2 x}{ds^2} + \left[ \frac{1}{\rho^2} + \frac{1}{(B\rho)} \frac{\partial B_y(s)}{\partial x} \right] x = 0
\]

\[
\frac{d^2 y}{dy^2} - \frac{1}{(B\rho)} \frac{\partial B_x(s)}{\partial y} y = 0
\]

This is in the form of a “Hill’s Equation”: \(x'' + K(s)x = 0\)

- the most general equation for small deviations from an ideal trajectory (first used to study stability of lunar orbit)

If \(K\) is constant, this is just a harmonic oscillator, and the solution is

\[
x(s) = A \sin\left(\sqrt{K} s + \delta\right)
\]

\(A\) and \(\delta\) determined by initial conditions

We therefore looks for a solution that looks “kinda sorta” like that...
General Solution: Betatron Motion

- We find (after a lot of algebra) that we can describe particle motion in terms of initial conditions and a “beta function” $\beta(s)$, which is only a function of location along the nominal path.

- In other words, particles undergo “pseudo-harmonic” motion about the nominal trajectory, with a variable wavelength.

- Note: $\beta$ has units of [length], so the amplitude has units of [length]^{1/2}

$$x(s) = A\sqrt{\beta(s)}\cos(\psi(s) + \delta)$$

$$\psi(s) = \int_{0}^{s} \frac{ds}{\beta(s)}$$

The “betatron function” $\beta(s)$ is effectively the local wavenumber and also defines the beam envelope.

Lateral deviation in one plane

Phase advance

follows periodicity of machine
Symmetric Treatment of FODO Cell

- We generally evaluate the FODO cell center of the focusing quad, it looks like, which makes the problem symmetric. Example: Old FNAL Main Ring
Beam Line Calculation: MAD

- We could calculate the lattice functions by hand, but...
- There have been and continue to be countless accelerator modeling programs; however MAD ("Methodical Accelerator Design"), started in 1990, continues to be the "Lingua Franca"

```
main_ring.madx

! One FODO cell from the FNAL Main Ring (NAL Design Report, 1968)
! beam, particle=proton,energy=400.938272,npart=1.0E9;

LQ:=1.067;
LD:=29.74-2*LQ;
qf: QUADRUPOLE, L=LQ, K1=.0195;
d: DRIFT, L=LD;
qd: QUADRUPOLE, L=LQ, K1=-.0195;
fodo: line = (qf,d,qd,qd,d,qf);
use, period=fodo;
match, sequence=FODO;
SELECT, FLAG=SECTORMAP, clear;
SELECT, FLAG=TWISS, column=name,s,betx,alfx,bety,alfy,mux,muy;
TWISS, SAVE;
PLOT, interpolate=true,,colour=100,HAXIS=S, VAXIS1=BETX,BETY;
PLOT, interpolate=true,,colour=100,HAXIS=S, VAXIS1=ALFX,ALFY;
stop;
```

- half quad K1=1/(2f)
- build FODO cell
- force periodicity
- calculate Twiss parameters

98.4m (exact) vs. 99.4m (thin lens)

- 24.7m vs. 26.4m
It’s important to remember that the betatron function represents a bounding envelope to the beam motion, not the beam motion itself.

Normalized particle trajectory

Trajectories over multiple turns (or trajectories of multiple particles!)

\[
x(s) = A[\beta(s)]^{1/2} \sin(\psi(s) + \delta)
\]

\[
\psi(s) = \int_0^s \frac{ds}{\beta(s)}
\]

\(\beta(s)\) is also effectively the local wave number which determines the rate of phase advance.

Closely spaced strong quads ➔ small \(\beta\ ➔ \) small aperture, lots of wiggles

Sparsely spaced weak quads ➔ large \(\beta\ ➔ \) large aperture, few wiggles
A particle returning to the same point over many terms traces an ellipse, defined by the “beta function”, $\beta$, and two additional lattice parameters, $\alpha$ and $\gamma$.

$$\beta x'^2 + 2\alpha xx' + \gamma x^2 = A^2 = \text{constant}$$

$$\alpha = -\frac{1}{2} \frac{d\beta}{ds} \quad \gamma = \frac{1+\alpha^2}{\beta}$$

$\alpha$, $\beta$, and $\gamma$ = “Twiss Parameters” - NOT to be confused with relativistic $\beta$ and $\gamma$!

An ensemble of particles can characterized by a bounding ellipse, known as the “emittance”

- Definitions vary: RMS, 95%, 99%, etc

$$\beta x' + 2\alpha xx' + \gamma x^2 = \epsilon$$

Units of length

Area = $\epsilon \pi$
If we use the Gaussian definition emittance, then the beam size is

$$\sigma_x = \sqrt{\beta_x \epsilon}$$

Emittance is constant at a constant energy, but as particles accelerate, the emittance decreases

$$\epsilon \propto \frac{1}{\beta \gamma}$$

This is known as “adiabatic damping”. We therefore define a “normalized emittance”

$$\epsilon_N \equiv \beta \gamma \epsilon$$

which is constant with energy. Thus, at a particular energy

$$\sigma_x = \sqrt{\beta_x \epsilon_N} \propto \frac{1}{\sqrt{p}}$$

Might be a factor here; e.g. 6 for 95% emittance

Relativistic $\beta$ and $\gamma$ (yes, I know it’s confusing)
As we go through a lattice the shape in phase space varies, by the bounding emittance remains constant.

- \( \beta = \max \)  
  \( \alpha = 0 \)  
  \( \Rightarrow \) maximum

- \( \beta = \) decreasing  
  \( \alpha > 0 \)  
  \( \Rightarrow \) focusing

- \( \beta = \min \)  
  \( \alpha = 0 \)  
  \( \Rightarrow \) minimum

- \( \beta = \) increasing  
  \( \alpha < 0 \)  
  \( \Rightarrow \) defocusing

large spatial distribution  
small angular distribution

small spatial distribution  
large angular distribution
As particles go around a ring, they will undergo a number of betatrons oscillations $\nu$ (sometimes $Q$) given by

$$\nu = \frac{1}{2\pi} \int \frac{ds}{\beta(s)}$$

This is referred to as the “tune”.

We can generally think of the tune in two parts:

- Integer: magnet/aperture optimization
- Fraction: Beam Stability
If the tune is an integer, or low order rational number, then the effect of any imperfection or perturbation will tend be reinforced on subsequent orbits.

When we add the effects of coupling between the planes, we find this is also true for combinations of the tunes from both planes, so in general, we want to avoid

\[ k_x \nu_x \pm k_y \nu_y = \text{integer} \Rightarrow \text{(resonant instability)} \]

“small” integers

\[ \Rightarrow \text{Avoid lines in the “tune plane”} \]

Many instabilities occur when something perturbs the tune of the beam, or part of the beam, until it falls onto a resonance, thus you will often hear effects characterized by the “tune shift” they produce.

- For example: the maximum tune shift sets the absolute luminosity limit in a collider (more about this in a bit...)

E. Prebys, Hadron Colliders, Lecture 1
Off-Momentum Particles

- Our previous discussion implicitly assumed that all particles were at the same momentum
  - Each quad has a constant focal length
  - There is a single nominal trajectory

- In practice, this is never true. Particles will have a distribution about the nominal momentum, typically ~.1% or so.

- We will characterize the behavior of off-momentum particles in the following ways
  - “Dispersion” ($D$): the dependence of position on deviations from the nominal momentum
    \[
    \Delta x(s) = D_x(s) \frac{\Delta p}{p_0}
    \]
    Dispersion has units of length
    
    $D$ has units of length
  - “Chromaticity” ($\eta$): the change in the tune caused by the different focal lengths for off-momentum particles (the focal length goes up with momentum)
    \[
    \Delta \nu_x = \xi_x \frac{\Delta p}{p_0}
    \]
    (sometimes $\frac{\Delta \nu_x}{\nu_x} = \xi_x \frac{\Delta p}{p_0}$)
  - Path length changes (“momentum compaction”)
    \[
    \frac{\Delta L}{L} = \alpha_c \frac{\Delta p}{p}
    \]
    We overloaded $\beta$ and $\gamma$; might as well overload $\alpha$, too.
The chromaticity ($\xi$) and the momentum compaction ($\alpha$) are properties of the entire ring. However, the dispersion ($D(s)$) is another position dependent lattice function, which follows the periodicity of the machine.

If we look at our standard FODO cell, but include the bend magnets, we find that the dispersion functions track the beta functions.

Typically dispersion is ~meters and momentum spread is ~.1%, so motion due to dispersion is ~mm.
Insertions

- So far, we’ve talked about nice, periodic lattice, but that may not be all that useful in the real world. In particular, we generally want
  - Locations for injection of extraction.
  - “Straight” sections for RF, instrumentation, etc
  - Low beta points for collisions

- Since we generally think of these as taking the place of things in our lattice, we call them “insertions”

```
FODO  FODO  FODO  Insertion  FODO  FODO  FODO
```

Match lattice functions
Simply modifying a section of the lattice without matching will result in a distortion of the lattice functions around the ring (sometimes called “beta beating”)

Here’s an example of increasing the drift space in one FODO cell from 5 to 7.5 m
A Collins insertion is an arrangement of magnets designed to insert a straight section, while matching lattice parameters. It’s also necessary to modify the dipoles on either side of the straight section to “suppress” dispersion.
In experimental applications, we will often want to focus beam down to a waist (minimum $\beta$) in both planes. In general, we can accomplish this with a triplet of quadrupoles.

Such triplets are a workhorse in beam lines, and you’ll see them wherever you want to focus beam down to a point.

- Can also be used to match lattice functions between dissimilar beam line segments

The solution, starting with a arbitrary lattice functions, is not trivial and in general these problems are solved numerically (eg, MAD can do this)
Low $\beta$ Insertions

- In a collider, we will want to focus the beam in both planes as small as possible.
- This can be done with a symmetric pair of focusing triplets, matched to the lattice functions (dispersion suppression is assumed)

\[ \beta(s) = \beta^* + \frac{s^2}{\beta^*} \]

where $s$ is measured from the location of the waist

- Near the focus, $\beta$ evolves as $\beta = \beta^*; \alpha = 0$
- This means that the smaller I make $\beta^*$, the bigger the beam gets in the focusing triplets!
  - We’ll discuss this much more shortly
Behavior near low-β insertion

\[ \beta(\Delta s) = \beta^* + \frac{\Delta s^2}{\beta^*} \]

- small \( \beta^* \) means large \( \beta \)
  (aperture) at focusing triplet

Much more about this shortly!
In our definition and derivation of the lattice function, a closed path through a periodic system. This definition doesn’t exist for a beam line, but once we know the lattice functions at one point, we know how to propagate the lattice function down the beam line.

\[ M(\text{out}, \text{in}) \]
When extracting beam from a ring, the initial optics of the beam line are set by the optics at the point of extraction.

\[
\begin{pmatrix}
\alpha_{in} \\
\beta_{in} \\
\gamma_{in}
\end{pmatrix}
\]

For particles from a source, the initial lattice functions can be defined by the distribution of the particles out of the source.

\[
\begin{pmatrix}
\alpha_{in} \\
\beta_{in} \\
\gamma_{in}
\end{pmatrix}
\]