Your Name:____________________________________

USPAS Fundamentals of Accelerator Physics Final Exam

June 15, 2018

General Guidelines - Please Read!

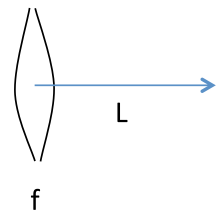
- REMEMBER TO WRITE YOUR NAME above!
- This is an "open book" exam. You may use the texts, lectures, homework, or any of the online resources found at the course website (`http://home.fnal.gov/~prebys/misc/uspas_2018/`), including the previous final. You may use your computer (Excel, MathCad, etc) to do calculations; however, you are expected to work independently and to not seek out other online sources for the solutions.
- The exam consists of 12 questions, some of which have multiple parts. Not all parts are of equal difficulty and not all have equal weight.
- You may use any equations that appeared in the lectures, textbook or assigned homework, without re-deriving them.
- You will need to work out some results on your computer or scratch paper, but **your final answer should be written on the exam itself!**
- You are not required to show your work, but if you include the key equations for any derivation, you may receive partial credit even if the final answer is incorrect. There is no need to show your work or explain your reasoning for multiple choice and yes/no answers.
- If the question specifies units, you can use the appropriate prefix to put the answer in a reasonable range, e.g. eV→MeV.
- All problems are straightforward applications of what you have learned. There are no trick questions or complex calculations. If you find yourself doing a lot of work, it's a good sign you're not doing the problem correctly. Stop and think!

1. Please answer the following questions for a proton ($m_pc^2 = .938$GeV) with a kinetic energy of 1 GeV.
   (a) What is its momentum [GeV/c]?

   (b) What is its velocity [m/s]?

   (c) What magnetic field will cause it to travel with a 2 m radius of curvature [T]?

2. We have a simple cell consisting of a quad that is focusing in x (focal length f), followed by a drift of length L, as shown.



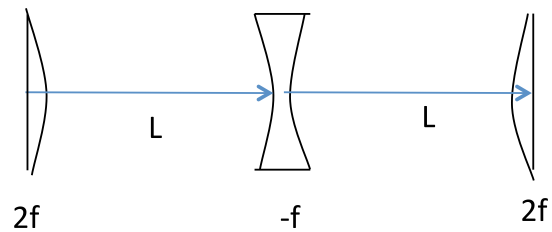
(a) Write a 2x2 transfer matrix for this cell in the x plane for a particle arriving from the left?

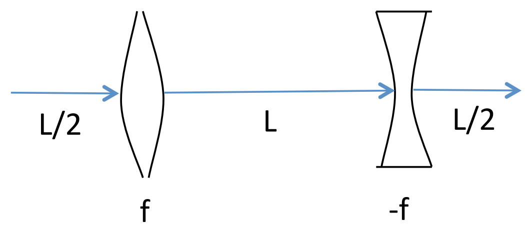
(b) Would stable periodic motion be possible with such cells *in the x plane only*? Explain why or why not.

(c) Write a 2x2 transfer matrix for this cell in the y plane for a particle arriving from the left?

(d) Would stable periodic motion be possible with such cells *in the y plane only*? Explain why or why not.

3. Consider the following two fundamental periodic cells, which have the same values for f and L





Circle *all* of the following items from among (a-d) that are *the same* for these two cells.

(a) The condition on the relationship between L and f for stable motion to exist.

(b) The phase advance μ of the cell.

(c) The lattice parameters α, β and γ at the ends of the cell.

(d) The lattice parameters α, β and γ at the center of defocusing quad.

4. A stable periodic cell of a synchrotron is described numerically (with units) by the matrix

$$\begin{pmatrix} 1 & 10\text{m} \\ -.05\text{m}^{-1} & .5 \end{pmatrix}$$

(a) What is the phase advance μ of the cell [radians]?

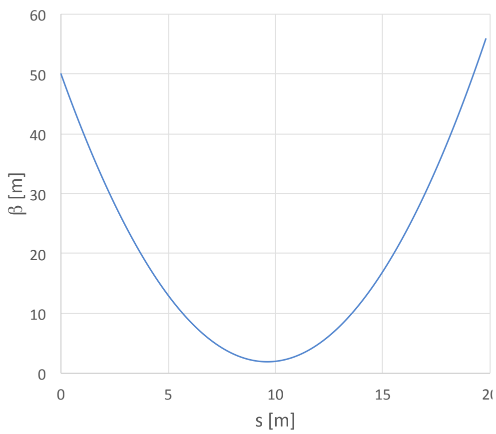
(b) What are the lattice parameters α and β [m] at the ends of the cell?

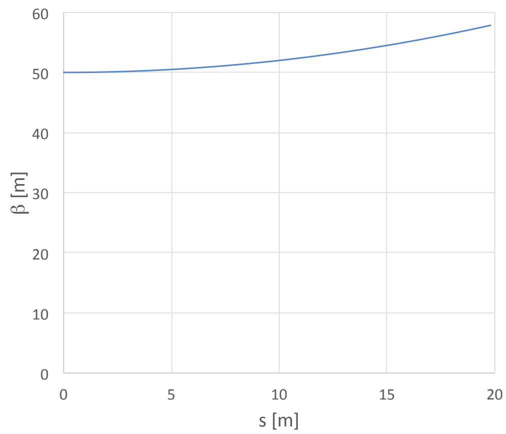
5. Below are plots of the evolution of the β function and phase advance ψ through a 20 m drift section. Clearly label each of the six with (a), (b), or (c), according to which if the following initial conditions it corresponds to

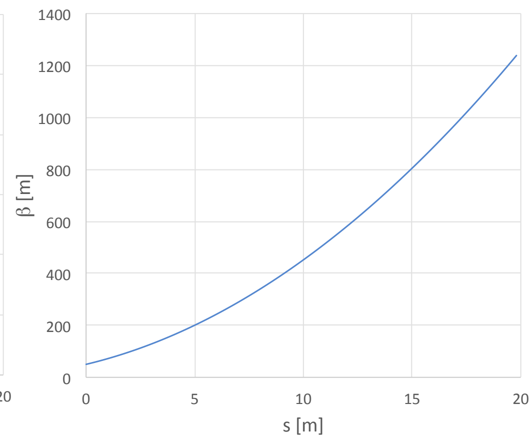
   (a) β_0=50 m, α_0=0.

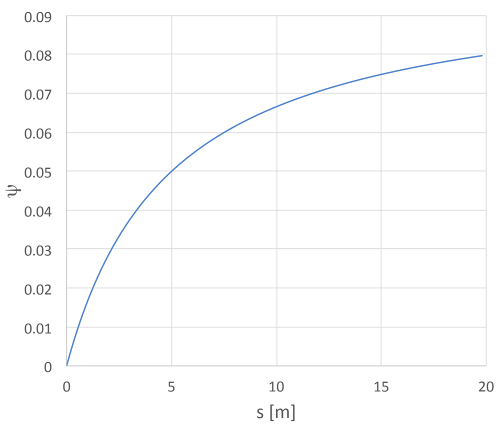
   (b) β_0=50 m, α_0=5.

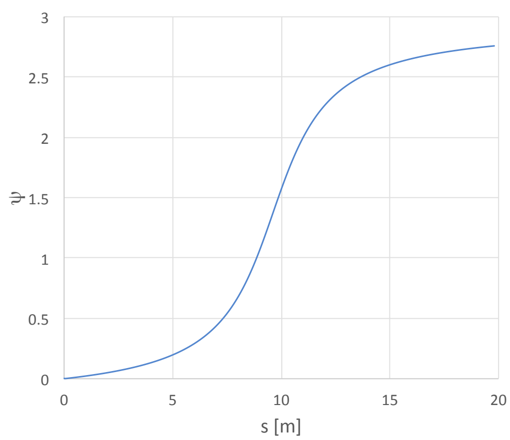
   (c) β_0=50 m, α_0=-10.

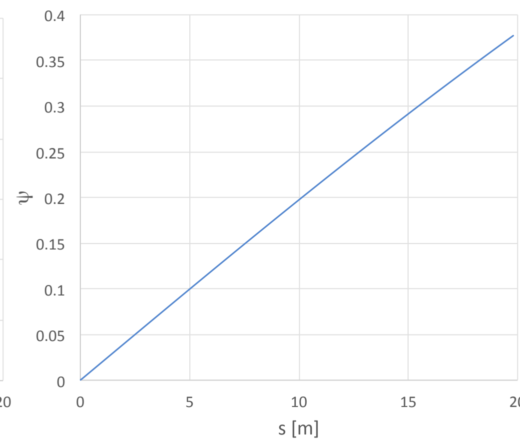












6. A properly matched proton beam with kinetic energy $K = 5$ GeV and normalized RMS emittance $\epsilon_N = 1\mu$m is circulating in a synchrotron with a maximum betatron function in the x plane of $\beta_{max} = 20$m.

   (a) What is the unnormalized (actual) emittance of the beam at this energy [μm]?

   (b) What is the maximum RMS beam size σ_x [mm]?

7. Assuming a beam distribution is described by a Gaussian emittance and is properly matched to the lattice, circle *all* the following statements which are *always* true for lattice functions and particles going through a long drift (straight region with no magnets), regardless of the initial conditions:
   (a) The β_x function will remain constant.
   (b) The β_x function will have a minimum.
   (c) If the drift region is long enough, the β_x function will eventually increase.
   (d) The α_x function will remain constant.
   (e) If the drift region is long enough, the sign of the α_x function will eventually change.
   (f) If the drift region is long enough, the α_x function will ultimately be negative.
   (g) The γ_x function will remain constant.
   (h) If the drift region is long enough, the γ_x function will eventually increase.
   (i) The RMS size of the spatial distribution σ_x will remain unchanged.
   (j) The RMS size of the angular distribution $\sigma_{x'}$ will remain unchanged.

8. Although it's not practical to build a circular e^+e^- collider at the energy frontier, it has been proposed to build a "Higgs Factory" in the tunnel that will eventually be used for the 100 TeV FCC proton collider. It will have approximately the following parameters:

   - (Electron) Beam energy: 120 GeV.
   - Circumference: $C = 100$ km.
   - Bend radius: $\rho_0 = 14$ km.
   - Beam Current: $I = 30$ mA.

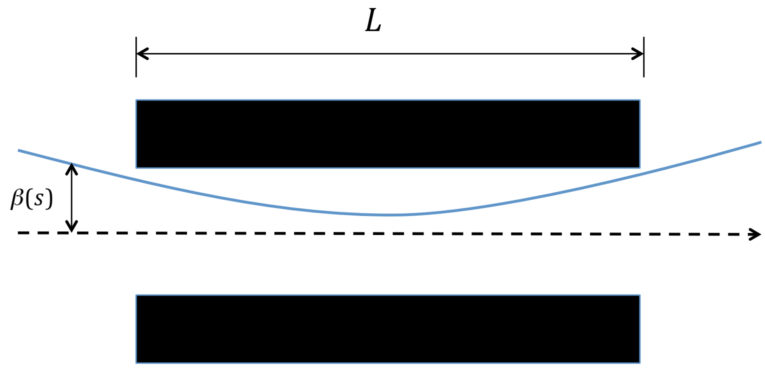
   Answer the following questions:

   (a) What will be the energy lost per turn [eV]?

   (b) What will be the damping time in x [s]?

   (c) What will be the total power lost to synchrotron radiation [W]?

9. Suppose we want to get beam through a collimator of length L that has a constant gap (aperture) along its length.



In this case, we clearly want to minimize the β function at the *ends* of the collimator.

(a) If β^* is the beta function at the middle of of the collimator, what will be the β function at the ends, in terms of β^* and L?

(b) What value of β^* will minimize the value of β at the ends?

(c) What will be this minimum β at the ends, in terms of L?

10. The future that wasn't: the Superconducting Super Collider (SSC) was a was a proposed proton-proton collider in the US in the 1990s. It had the following parameters related to the RF system:

- Circumference: $C = 87$ km.
- Injection kinetic Energy: K_0 =2 TeV
- Maximum kinetic Energy: K_1 =20 TeV
- Total (linear) ramp time: $T = 1500$ s
- Total RF voltage per turn: $V_{RF} = 8$ MV
- Harmonic of RF system: h =100,000
- Transition gamma: $\gamma_t = 105$ (i.e., beam injected above transition)

Evaluate the following just as the beam begins accelerating. You may assume the beam is ultra-relativistic ($v \approx c$):

(a) What is the frequency of the RF system [Hz]?

(b) What is the the stable synchronous phase angle [degrees]?

(c) What is the slip factor η?

(d) What is the longitudinal beta function β_L?

(e) What is the longitudinal tune ν_s for small amplitude oscillations?

(f) What is the maximum bucket height ΔE_b [eV]?

(g) If the RMS of the energy distribution σ_E is 1/4 of the bucket height, what is the RMS of the time distribution σ_t [s]?

11. A proton beam with kinetic energy of 200 MeV has an RMS *energy* spread of .1%. If it's circulating in a synchrotron with a maximum dispersion of $D_x = 5$m, what would be the RMS σ_x resulting from this energy spread [m]?

12. In the lab, you learned about the following types of hardware:
    - Quadrupole magnets
    - Dipole magnets
    - Resonant cavities
    - BPMs
    - Toroids
    - Resistive wall monitors.

    Explain briefly how you could use two of these to measure the energy of a beam of known particles at the end of a beam line. You need only concern yourself with the classes of device. You may assume the parameters are appropriate to the task at hand.

END OF EXAM