SYNCHROTRON RADIATION AND LIGHT SOURCES

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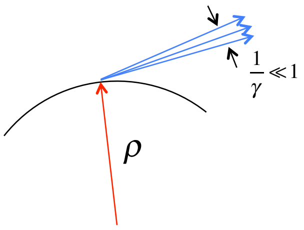
Synchrotron Radiation

For a relativistic particle, the total radiated power (S&E 8.1) is



$$a = \text{acceleration} = \frac{v^2}{\rho} \approx \frac{c^2}{\rho}$$

$$P = \frac{1}{6\pi\epsilon_0}\frac{e^2a^2}{c^3}\gamma^4$$

$$\boxed{\approx \frac{1}{6\pi\epsilon_0}\frac{e^2c}{\rho^2}\gamma^4} = \frac{1}{6\pi\epsilon_0}\frac{e^2c}{\rho^2}\left(\frac{E}{m_0c^2}\right)^4$$

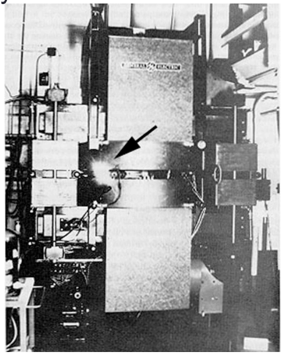
Electron radiates 10^{13} times more than a proton of the same energy!

In a magnetic field

$$\rho = \frac{m\gamma c}{eB} \quad \rightarrow P \quad \boxed{\begin{aligned} &= \frac{e^4}{6\pi\epsilon_0}\frac{B^2}{m_0^2c}\gamma^2 \\ &= \frac{e^4}{6\pi\epsilon_0 m^4c^5}B^2E^2 \end{aligned}}$$

First Observation of Synchrotron Radiation

- The first attempt to observe synchrotron radiation was in 1944 at the 100 MeV GE betatron
  - Because of a miscalculation, they were looking in the microwave region rather than the visible (in fact the walls were opaque), so although the say an energy decay, they did not observe the radiation.
- Synchrotron radiation was first successfully observed in 1947 by Elder, Gurewitsch, and Langmuir at the GE 70 MeV electron synchrotron.



Effects of Synchrotron Radiation

- Two competing effects
  - Damping

energy

$$\tau_{\Delta E} \propto \tau \frac{E_s}{U_s}$$

damping time

period

energy lost per turn

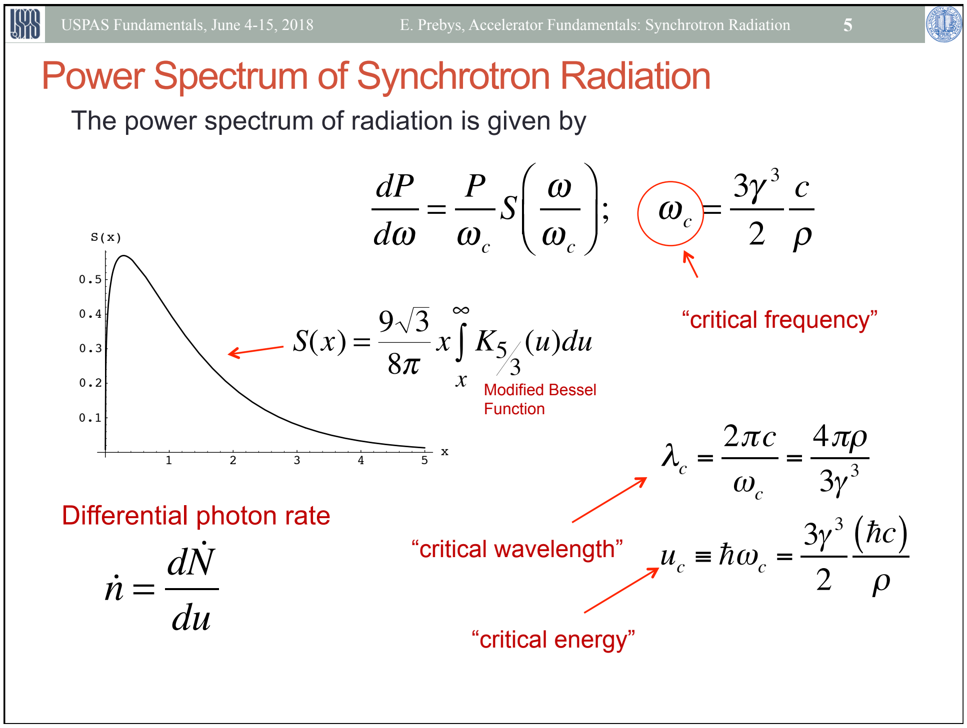
  - Quantum “heating” effects related to the statistics of the photons

$$N_p = \dot{N}\tau \quad \rightarrow \quad \sigma_{\Delta E} = \sqrt{\dot{N}\tau_{\Delta E}\left\langle u^2 \right\rangle}$$

Number of photons per period

Rate of photon emission

Average photon energy



Power Spectrum of Synchrotron Radiation

The power spectrum of radiation is given by

$$\frac{dP}{d\omega} = \frac{P}{\omega_c} S\left(\frac{\omega}{\omega_c}\right); \quad \omega_c = \frac{3\gamma^3}{2}\frac{c}{\rho}$$

"critical frequency"

$$S(x) = \frac{9\sqrt{3}}{8\pi} x \int_x^\infty K_{5/3}(u)du$$

Modified Bessel Function

$$\lambda_c = \frac{2\pi c}{\omega_c} = \frac{4\pi\rho}{3\gamma^3}$$

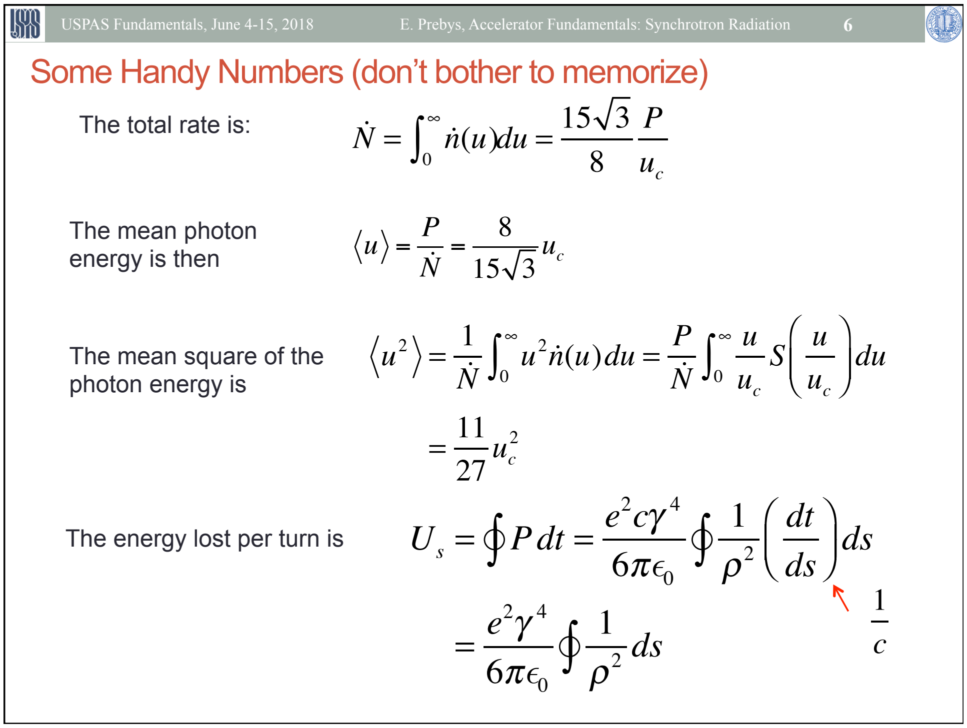
"critical wavelength"

$$u_c \equiv \hbar\omega_c = \frac{3\gamma^3}{2}\frac{(\hbar c)}{\rho}$$

"critical energy"

Differential photon rate

$$\dot{n} = \frac{d\dot{N}}{du}$$



Some Handy Numbers (don't bother to memorize)

The total rate is:

$$\dot{N} = \int_0^\infty \dot{n}(u)du = \frac{15\sqrt{3}}{8}\frac{P}{u_c}$$

The mean photon energy is then

$$\langle u \rangle = \frac{P}{\dot{N}} = \frac{8}{15\sqrt{3}} u_c$$

The mean square of the photon energy is

$$\langle u^2 \rangle = \frac{1}{\dot{N}} \int_0^\infty u^2 \dot{n}(u) du = \frac{P}{\dot{N}} \int_0^\infty \frac{u}{u_c} S\left(\frac{u}{u_c}\right) du$$

$$= \frac{11}{27} u_c^2$$

The energy lost per turn is

$$U_s = \oint P\,dt = \frac{e^2 c\gamma^4}{6\pi\epsilon_0} \oint \frac{1}{\rho^2}\left(\frac{dt}{ds}\right) ds$$

$$\frac{1}{c}$$

$$= \frac{e^2\gamma^4}{6\pi\epsilon_0} \oint \frac{1}{\rho^2} ds$$

Example: The Failed Experiment

- In 1944 GE looked for synchrotron radiation in a 100 MeV electron beam.
  - Assume B=1T
- We have
  - E≈pc=100 MeV
  - mc^2=.511 MeV
  - γ=E/(mc^2) = 196
  - (Bρ)=100/300=.333 T-m
  - ρ=(Bρ)/B=.333 m

eV-m

$$u_c = \frac{3\gamma^3}{2}\frac{(\hbar c)}{\rho} = \frac{3(196)^3(1.97\times 10^{-7})}{2(.333)} = 6.6 \text{ eV}$$

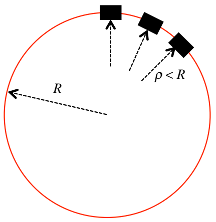
$$\langle u\rangle = \frac{8}{15\sqrt{3}}u_c = 2.05 \text{ eV}$$

eV-μm

$$\lambda_{\langle u\rangle} = \frac{(hc)}{\langle u\rangle} = \frac{1.2}{2.05} = .587\ \mu\text{m}$$

Visible yellow light, NOT microwaves

It's important to remember that ρ is *not* the curvature of the accelerator as a whole, but rather the curvature of individual magnets.



$$\Delta\theta = \frac{\Delta s}{\rho} \rightarrow \oint \frac{ds}{\rho} = 2\pi$$

So if an accelerator is built using magnets of a fixed radius ρ$_0$, then the energy lost per turn is

$$U_s = \frac{e^2\gamma^4}{6\pi\epsilon_0}\oint\frac{1}{\rho^2}ds = \frac{e^2\gamma^4}{6\pi\epsilon_0\rho_0}\oint\frac{1}{\rho}ds = \boxed{\frac{e^2\gamma^4}{3\epsilon_0\rho_0}}$$

"isomagnetic"

For electrons

$$U_s[\text{MeV}] = .0885\frac{E^4[\text{GeV}]}{\rho_0[\text{m}]}$$

$$u_c = \hbar\omega_c = \frac{3\gamma^3\hbar}{2}\frac{c}{\rho_0}$$

$$u_c[\text{keV}] = 2.218\frac{E^3[\text{GeV}]}{\rho_0[\text{m}]}$$

$$N_s = \dot{N}\tau = \frac{15\sqrt{3}}{8}\frac{P}{u_c}\tau = \frac{15\sqrt{3}}{8}\frac{U_s}{u_c}$$

$$= .1296E[\text{GeV}]$$

photons/turn

Example: CESR

$$E = 5.29 \text{ GeV}$$

$$\rho_0 = 98 \text{ m}$$

$$U_s = .71 \text{ MeV}$$

$$\langle u\rangle = \frac{8}{15\sqrt{3}}u_c = .98 \text{ keV}$$

$$\sqrt{\langle u^2\rangle} = \sqrt{\frac{11}{27}}u_c = 2.0 \text{ keV}$$

$$\text{N}_s = 721$$

Effects of Synchrotron Radiation

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energy

$$\tau_{\Delta E} \propto \tau \frac{E_s}{U_s}$$

damping time

period

energy lost per turn

  - Quantum "heating" effects related to the statistics of the photons

$$N_p = \dot{N}\tau \quad \rightarrow \quad \sigma_{\Delta E} = \sqrt{\dot{N}\tau_{\Delta E}\left\langle u^2 \right\rangle}$$

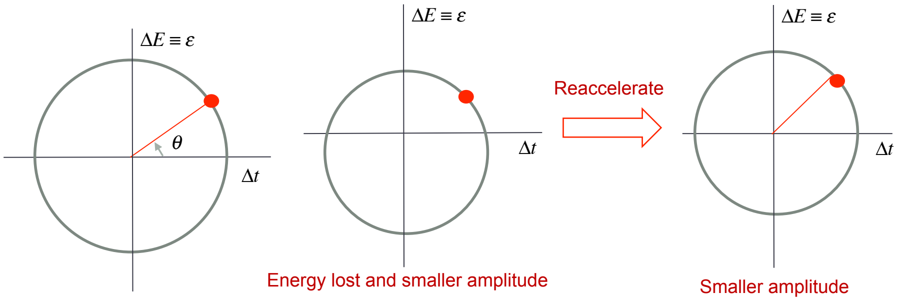
Number of photons per period

Rate of photon emission

Average photon energy

Small Amplitude Longitudinal Motion

$P \propto E^2 \rightarrow$ Particles lose more energy at the top of this cycle than the bottom



Energy lost and smaller amplitude

Smaller amplitude

$$\left\langle \frac{d\varepsilon_0^2}{dt} \right\rangle = \frac{1}{\tau_s} \oint \frac{d\varepsilon_0^2}{dt} dt$$

$$\boxed{\frac{d\varepsilon_0^2}{dt} = 2\varepsilon_0 \frac{d\varepsilon}{dt} = -2\varepsilon_0 P}$$

$$= -\frac{2}{\tau_s} \oint \left\langle \varepsilon P \right\rangle dt + \frac{1}{\tau_s} \oint \dot{N} \left\langle u^2 \right\rangle dt$$

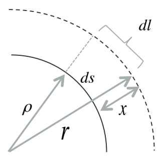
damping term

Heating term due to statistical fluctuations

Evaluate integral in damping term

$$\oint\langle\varepsilon P\rangle dt=\frac{1}{c}\oint\left(1+\frac{x}{\rho}\right)\langle\varepsilon P\rangle ds$$

$$\approx\frac{1}{c}\oint\left(1+D\frac{\varepsilon}{\rho E_s}\right)\langle\varepsilon P\rangle ds$$



$$dl=(\rho+x)ds=cdt$$

$$\rightarrow dt=\frac{1}{c}(\rho+x)ds$$

use $x=D\frac{\Delta p}{p}\approx D\frac{\varepsilon}{E_s}$

Recall

$$P=\frac{e^4}{6\pi\epsilon_0 m^4c^5}B^2E^2\rightarrow\frac{dP}{dE}=2P\left(\frac{1}{B_0}\frac{dB}{dE}+\frac{1}{E}\right)$$

$$P(\varepsilon)=P_s+\frac{dP}{dE}\varepsilon=P_s\left(1+2\left(\frac{1}{B_0}\frac{dB}{dE}+\frac{1}{E_s}\right)\varepsilon\right)$$

Can't ignore anything!!

Dependence of field

$$\frac{dB}{dx}=B'$$
$$=\kappa(B\rho)$$

$$\longrightarrow\frac{dB}{dE}=\frac{dB}{dx}\frac{dx}{dE}$$
$$=\frac{\kappa(B\rho)D}{E_s}$$

$$\longrightarrow P(\varepsilon)=P_s\left(1+\frac{2\varepsilon}{E_s}(\kappa\rho D+1)\right)$$

Putting it all together…

$$\oint\langle\varepsilon P\rangle dt=\frac{1}{c}\oint\left\langle\varepsilon P_s\left(1+\frac{\varepsilon}{E_s}\frac{D}{\rho}\right)\left(1+\frac{2\varepsilon}{E_s}(\kappa\rho D+1)\right)\right\rangle ds$$

$$=\frac{1}{c}\oint\left\langle P_s\left(\cancelto{0}{\varepsilon}+\frac{\varepsilon^2}{E_s}\left(2+2\kappa\rho D+\frac{D}{\rho}\right)+\cancelto{0}{\varepsilon^3}\frac{2D(\kappa\rho D+1)}{E_s\rho}\right)\right\rangle ds$$

$$=\frac{1}{c}\frac{\varepsilon_0^2}{2E_s}\oint P_s\left(2+2\kappa\rho D+\frac{D}{\rho}\right)ds$$

$$=\frac{\varepsilon_0^2U_s}{E_s}+\frac{\varepsilon_0^2}{2E_s}\frac{1}{c}\oint P_s\left(2\kappa\rho D+\frac{D}{\rho}\right)ds$$

$$=\frac{\varepsilon_0^2U_s}{E_s}+\frac{\varepsilon_0^2U_s}{2E_s}\mathcal{D}$$

$$\boxed{=\frac{\varepsilon_0^2U_s}{2E_s}(2+\mathcal{D})}$$

use

$$\varepsilon=\varepsilon_0\sin(2\pi\nu_s n+\delta)$$
$$\rightarrow\langle\varepsilon\rangle=\langle\varepsilon^3\rangle=0$$
$$\langle\varepsilon^2\rangle=\frac{\varepsilon_0^2}{2}$$

note $\frac{1}{c}\oint P_s\,ds=\frac{1}{c}(\text{const})\oint\frac{1}{\rho^2}ds$

$$=U_s$$

$$\frac{1}{c}\oint P_s\left(2\kappa\rho D+\frac{D}{\rho}\right)ds=\frac{1}{c}(\text{const})\oint\frac{1}{\rho^2}\left(2\kappa\rho D+\frac{D}{\rho}\right)ds$$

$$=U_s\mathcal{D}$$

where $\mathcal{D}\equiv\dfrac{\oint\frac{1}{\rho^2}\left(2\kappa\rho D+\frac{D}{\rho}\right)ds}{\oint\frac{1}{\rho^2}ds}$

Reminder: Damping + Heating

- In general, if I have a simple damping force of the form

$$\frac{dA}{dt} = -\lambda A$$

the solution is $A(t) = A_0 e^{-\lambda t} = A_0 e^{-t/\tau}$; where $\tau = 1/\lambda$

- If I add a constant heating term $\frac{dA}{dt} = -\lambda A + h$

$$\int \frac{dA}{A - h/\lambda} = \int -\lambda\, dt$$

$$\rightarrow \ln(A - h/\lambda) = -\lambda t + K$$

$$\rightarrow A = Ce^{-\lambda t} + h/\lambda$$

$$A(0) = A_0 \rightarrow C = 1 - h/\lambda$$

$$\boxed{\begin{aligned} &\rightarrow A(t) = A_0 e^{-\lambda t} + \frac{h}{\lambda}\left(1 - e^{-\lambda t}\right) \\ &\rightarrow A(\infty) = \frac{h}{\lambda} = h\tau \end{aligned}}$$

Result

$$\left\langle \frac{d\varepsilon_0^2}{dt} \right\rangle = -\frac{2}{\tau_s}\oint \langle \varepsilon P \rangle dt + \frac{1}{\tau_s}\oint \dot{N}\langle u^2 \rangle dt$$

$$= \underbrace{-\frac{\varepsilon_0^2 U_s}{\tau_s E_s}(2+\mathcal{D})}_{\text{damping}} + \underbrace{\frac{1}{\tau_s}\oint \dot{N}\langle u^2 \rangle dt}_{\text{heating}}$$

$$\boxed{\text{where } \mathcal{D} \equiv \frac{\oint \frac{1}{\rho^2}\left(2\kappa\rho D + \frac{D}{\rho}\right) ds}{\oint \frac{1}{\rho^2} ds}}$$

$$\varepsilon_0^2(t) = \varepsilon_0^2(0) e^{-t/\tau_{\varepsilon^2}} + \varepsilon_0^2(\infty)\left(1 - e^{-t/\tau_{\varepsilon^2}}\right)$$

where $\frac{1}{\tau_{\varepsilon^2}} = \frac{U_s}{\tau_s E_s}(2+\mathcal{D})$

$$\varepsilon_0^2(\infty) = \frac{\tau_{\varepsilon^2}}{\tau_s}\oint \dot{N}\langle u^2 \rangle dt$$

The energy then decays in a time

$$\tau_\varepsilon = 2\tau_{\varepsilon^2}$$

$$\boxed{\frac{1}{\tau_\varepsilon} = \frac{U_s}{2\tau_s E_s}(2+\mathcal{D})}$$

Longitudinal Damping in a "Normal" Synchrotron

- So far we have talked about "separated function", "isomagnetic" lattices, which has
  - A single type of dipole: $\kappa = 0; \rho = \rho_0$
  - Quadrupoles: $\kappa \neq 0; \rho = \infty$
- In this case

$$\mathcal{D} \equiv \frac{\oint \frac{1}{\rho^2}\left(2\kappa\rho D + \frac{D}{\rho}\right)ds}{\oint \frac{1}{\rho^2}ds} = \frac{\frac{1}{\rho_0^2}\oint \frac{D}{\rho_0}ds}{\frac{1}{\rho_0}\oint \frac{1}{\rho_0}ds} = \frac{\frac{1}{\rho_0^2}(C\alpha_C)}{\frac{1}{\rho_0}(2\pi)}$$

$$= \frac{C\alpha_C}{2\pi\rho_0} \approx \alpha_C \ll 1$$

$$\boxed{\frac{1}{\tau_\varepsilon} \approx \frac{U_s}{\tau_s E_s}}$$

probably the answer you would have guessed without doing any calculations.

Equilibrium Energy Spread

- We can relate the spread in energy to the peak of the square with

$$\sigma_\varepsilon^2 = \left\langle \varepsilon_0^2(\infty) \right\rangle = \frac{1}{2}\varepsilon_0^2(\infty)$$

$$= \frac{1}{2}\frac{\tau_{\varepsilon^2}}{\tau_s}\oint \left\langle \dot{N}u^2 \right\rangle dt = \frac{\tau_\varepsilon}{4\tau_s}\oint \left\langle \dot{N}u^2 \right\rangle dt = \frac{E_S}{2U_s(2+\mathcal{D})}\oint \left\langle \dot{N}u^2 \right\rangle dt$$

Use $P = \frac{1}{6\pi\epsilon_0}\frac{e^2 c}{\rho^2}\gamma^4, \dot{N} = \frac{15\sqrt{3}}{8}\frac{P}{u_c}, \quad \left\langle u^2 \right\rangle = \frac{11}{27}u_c^2, \quad u_c = \frac{3}{2}\frac{\hbar\gamma^3}{\rho}c$

$$\tau_\varepsilon = \tau_s \frac{2E_s}{U_s(2+\mathcal{D})}, U_s = \frac{e^2\gamma^4}{3\epsilon_0\rho_0}$$

- This leads to

$$\rightarrow \oint \langle \dot{N}u^2 \rangle dt = \frac{55}{16\sqrt{3}}\frac{e^2\hbar c\gamma^7}{6\pi\epsilon_0}\oint\frac{1}{\rho^3}ds$$

$$= \frac{55}{16\sqrt{3}}\frac{e^2(\hbar c)\gamma^7}{3\epsilon_0\rho_0^2}$$

$$\rightarrow \sigma_\varepsilon{}^2 = \frac{E_S}{2U_s(2+\mathcal{D})}\left(\frac{55}{16\sqrt{3}}\frac{e^2(\hbar c)\gamma^7}{3\epsilon_0\rho_0^2}\right)$$

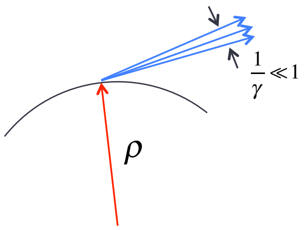
$$= \frac{E_S}{(1+\mathcal{D})}\frac{55}{32\sqrt{3}}\frac{(\hbar c)\gamma^3}{\rho_0}$$

$$= \frac{E_S}{(2+\mathcal{D})}\frac{55}{32\sqrt{3}}\frac{\hbar}{mc}\frac{(\gamma mc^2)}{\rho_0}\gamma^2$$

$$= \boxed{C_q\frac{\gamma^2 E_s^2}{(2+\mathcal{D})\rho_0}} \qquad C_q \equiv \frac{55}{32\sqrt{3}}\frac{\hbar}{mc} = 3.8\times10^{-13}\,\text{m} \;\; \text{(for electrons)}$$

Damping in the Vertical Plane

Synchrotron radiation

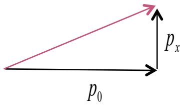


Energy lost along trajectory, so radiated power will reduce momentum along flight path

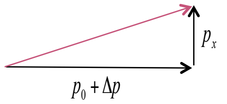
$$\frac{d\vec{p}}{dt} \approx -\frac{P}{c}\hat{\theta}$$

If we assume that the RF system restores the energy lost each turn, then

Energy lost along the path→ $\Delta y = \Delta y' = 0$
Energy restored along nominal path $\hat{s}$ ➔"adiabatic damping"



$$x' = \frac{p_x}{p_0}$$



Damping in the Vertical Plan (cont'd)

- The math follows much like the case of adiabatic damping, and we find that

$$\boxed{\frac{1}{\tau_y}=\frac{1}{2\tau_s}\frac{U_S}{E_S}=\frac{1}{2\tau_\varepsilon}}$$

- Unlike the longitudinal plane, there is no heating term, so in the absence of coupling, the emittance would damp to zero in the vertical plane.
    - This turns out to a problem for stability

Horizontal Plane

The horizontal plane has the same damping term as the vertical plane, but it has more contributions because the position depends on energy

betatron motion

ΔE

$$x=x_\beta+D\frac{\epsilon}{E_s}$$

$$x'=x'_\beta+D'\frac{\epsilon}{E_s}$$

where

$$x_\beta=a\sqrt{\beta}\cos\left(\psi(s)+\delta\right)\equiv a\sqrt{b}C$$

$$x'_\beta=-\frac{a}{\sqrt{\beta}}\left(\alpha\cos\left(\psi(s)+\delta\right)+\sin\left(\psi(s)+\delta\right)\right)\equiv-\frac{a}{\sqrt{\beta}}\left(\alpha C+S\right)$$

If we radiate a photon of energy *u*, it will change the energy, but not the position or the angle.

$$\Delta x=\left[\left(x_\beta+\Delta x_\beta\right)+D\frac{(\varepsilon-u)}{E_s}\right]-\left[x_\beta+D\frac{\varepsilon}{E_s}\right]$$

$$=\Delta x_\beta-D\frac{u}{E_s}=0$$

$$\longrightarrow\ \Delta x_\beta=D\frac{u}{E_s}$$

$$\Delta x'=\Delta x'_\beta-D'\frac{u}{E_s}=0$$

$$\longrightarrow\ \Delta x'_\beta=D'\frac{u}{E_s}$$

Result in Horizontal Plane

- Skipping a lot of math, we get

$$\boxed{\begin{aligned}\frac{1}{\tau_x} &= \frac{U_s}{2\tau_s E_s}\left(1-\mathcal{D}\right)\\ &\approx \frac{U_s}{2\tau_s E_s}\end{aligned}}$$

$$\text{where } \mathcal{D} \equiv \frac{\oint \frac{1}{\rho^2}\left(2\kappa\rho D + \frac{D}{\rho}\right)ds}{\oint \frac{1}{\rho^2}ds}$$

Same as longitudinal plane

Separated function isomagnetic synchrotrons

Equilibrium Emittance in X

- The equilibrium emittance is given by

$$\epsilon_x(\infty) = C_q \frac{\gamma^2}{(1-\mathcal{D})}\frac{\oint \frac{\mathcal{H}}{\rho^3}ds}{\oint \frac{1}{\rho^2}ds}$$

$$\text{where } C_q \equiv \frac{55}{32\sqrt{3}}\frac{\hbar}{mc} = 3.8\times 10^{-13}\text{m} \quad \text{(for electrons)}$$

$$\text{where } \mathcal{D} \equiv \frac{\oint \frac{1}{\rho^2}\left(2\kappa\rho D + \frac{D}{\rho}\right)ds}{\oint \frac{1}{\rho^2}ds}$$

$$\mathcal{H} \equiv \gamma D^2 + 2\alpha D D' + \beta D'^2$$

- For a separated function, isomagnetic machine, this becomes

$$\boxed{\epsilon_x(\infty) = C_q \frac{\gamma^2}{2\pi\rho_0(1-\mathcal{D})}\oint \frac{\mathcal{H}}{\rho}ds}$$

- With some handwaving, this can be approximated by

$$\boxed{\epsilon_x(\infty) \approx C_q \gamma^2 \frac{R}{\rho_0}\frac{1}{\nu_x^3}}$$

Robinson's Theorem

- Note:

$$\begin{aligned}\frac{1}{\tau_\varepsilon}+\frac{1}{\tau_x}+\frac{1}{\tau_y} &= \frac{U_s}{2\tau_s E_s}(2+\mathcal{D}) \\ &+\frac{U_s}{2\tau_s E_s}(1-\mathcal{D}) \\ &+\frac{U_s}{2\tau_s E_s} \\ &=\frac{2U_s}{\tau_s E_s}\end{aligned}$$

- This is called Robinson's theorem and it's *always* true. For a separated function, isomagnetic lattice, it simplifies to

$$\boxed{\begin{aligned}\frac{1}{\tau_\varepsilon} &= \frac{U_s}{\tau_s E_s} \\ \frac{1}{\tau_x}=\frac{1}{\tau_y} &= \frac{U_s}{2\tau_s E_s}\end{aligned}}$$

Cheat Sheet Summary

- For a separated function, isomagnetic synchrotron

Energy lost per turn → $U_s = \frac{e^2\gamma^4}{3\epsilon_0\rho_0}$; for electrons $U_s[\text{MeV}] = .0885\frac{E^4[\text{GeV}]}{\rho_0[\text{m}]}$

Longitudinal damping time → $\tau_\varepsilon \approx \tau_s\frac{E_S}{U_S}$

Transverse damping times → $\tau_x \approx 2\tau_s\frac{E_S}{U_S}$ → $\tau_y \approx \tau_x$

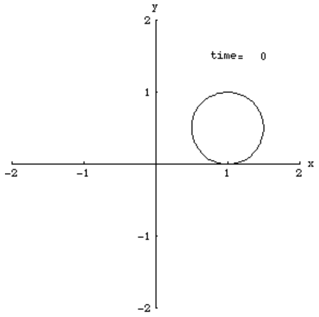
$\frac{1}{\tau_\varepsilon}+\frac{1}{\tau_x}+\frac{1}{\tau_y}=\frac{2U_S}{\tau_s E_S}$ ← Robinson's Theorem (always true)

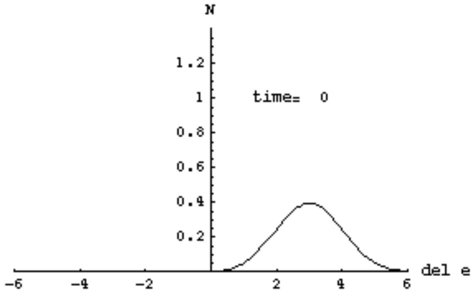
Equilibrium energy spread → $\sigma_\varepsilon{}^2(\infty) \approx C_q\frac{\gamma^2 E_s^2}{2\rho_0}$; for electrons $C_q \equiv \frac{55}{32\sqrt{3}}\frac{\hbar}{mc} = 3.8\times 10^{-13}\text{m}$

Equilibrium horizontal emittance → $\epsilon_x(\infty) \approx C_q\gamma^2\frac{R}{\rho_0}\frac{1}{\nu_x^3}$

Benefits of Damping

- Can inject off orbit and beam will damp down to equilibrium
  - Don't have to worry about painting or charge exchange like protons.
  - Can inject over many turns, or even continuously.
- Beams will naturally "cool" (i.e. reduce their emittance in phase space)
- Example: Beams injected off orbit into CESR





Considerations for e^+e^- Colliders

- In the case of proton-proton and proton-antiproton colliders, we assumed
  - The optics were the same in the two planes
  - The emittances were the same in the two planes
  - The normalized emittance was preserved.
- This allowed us to write

$$L = f\frac{N_b{}^2}{4\pi\sigma^2} = f_{rev}\frac{1}{4\pi}n_b N_b^2\frac{\gamma}{\beta^*\epsilon_N}$$

- In general, *none* of this will be true for e^+e^- colliders.
  - The emittance will be much smaller in the *y* plane
  - Because the emittance is large in the *x* plane, we will not be able to "squeeze" the optics as far without hitting the aperture in the focusing triplet, so in general, $\beta^*_x > \beta^*_y$.
- We must write

$$\boxed{L = f\frac{N_1N_2}{4\pi\sigma_x\sigma_y} = f_{rev}\frac{1}{4\pi}n_b\frac{N_1N_2}{\sqrt{\beta_x^*\epsilon_x\beta_y^*\epsilon_y}}}$$

Unnormalized(!) emittance

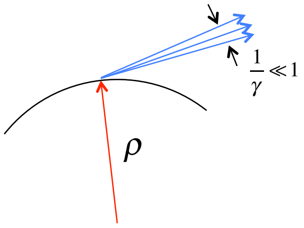
Synchrotron Light Sources

- Shortly after the discovery of synchrotron radiation, it was realized that the intense light that was produced could be used for many things
  - Radiography
  - Crystallography
  - Protein dynamics
  - …
- The first “light sources” were parasitic on electron machines that were primarily used for other things.
- As the demand grew, dedicated light sources began to emerge
- The figure or merit is the “brightness”

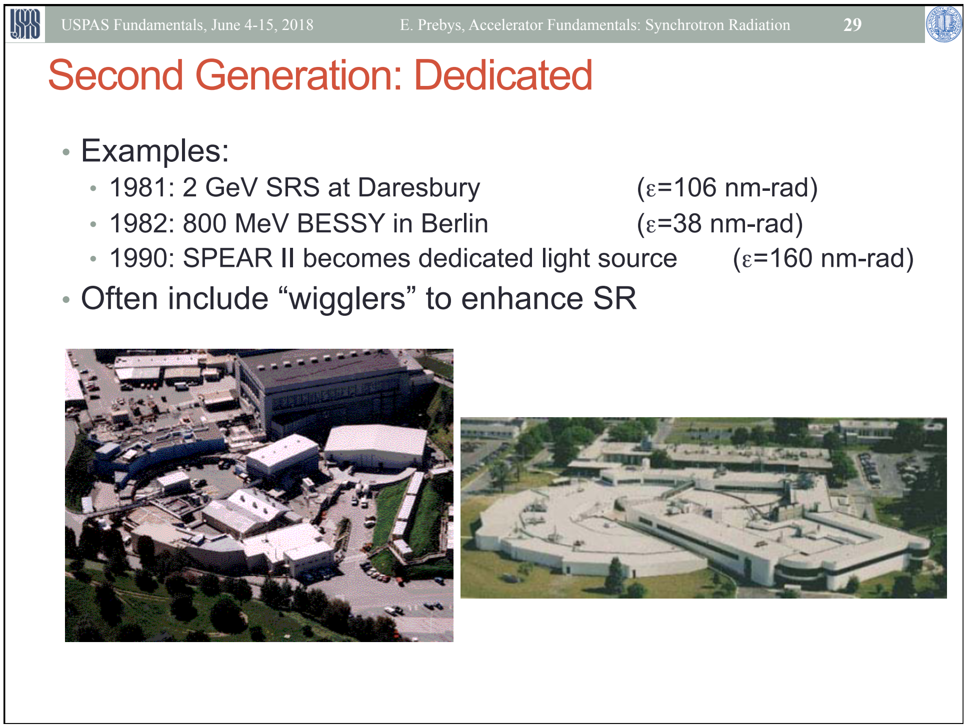
$$\text{photons/s/mm}^2 \,/\, \text{mrad}^2 \,/\, (\text{bandwidth})$$

First Generation: Parasitic Operations

- These just used the parasitic synchrotron light produced by the bend dipoles

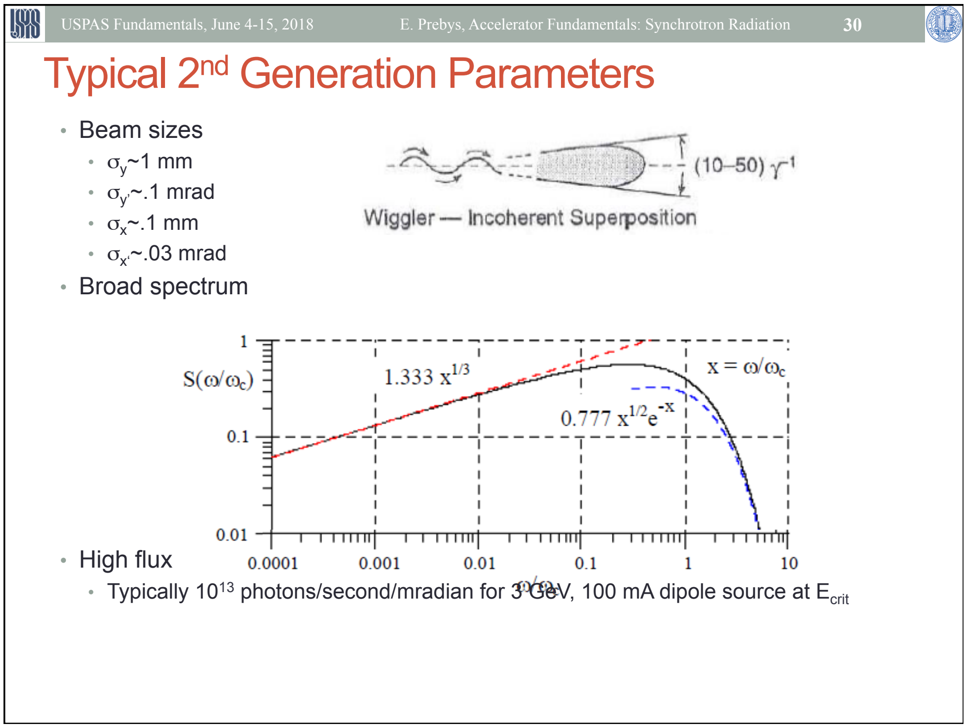


- Examples
  - SURF (1961): 180 MeV UV synchrotron at NBS
  - CESR (CHESS, 70’s): 6 GeV synchrotron at Cornell
  - Numerous others
- Typically large emittances, which limited brightness of the beam



Second Generation: Dedicated

- Examples:
  - 1981: 2 GeV SRS at Daresbury (ε=106 nm-rad)
  - 1982: 800 MeV BESSY in Berlin (ε=38 nm-rad)
  - 1990: SPEAR II becomes dedicated light source (ε=160 nm-rad)
- Often include “wigglers” to enhance SR

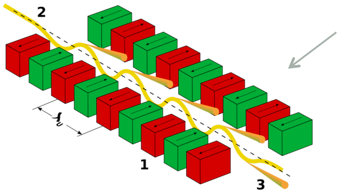


Typical 2nd Generation Parameters

- Beam sizes
  - σ_y~1 mm
  - $\sigma_{y'}$~.1 mrad
  - σ_x~.1 mm
  - $\sigma_{x'}$~.03 mrad
- Broad spectrum

- High flux
  - Typically 10^{13} photons/second/mradian for 3 GeV, 100 mA dipole source at E_{crit}

Undulators



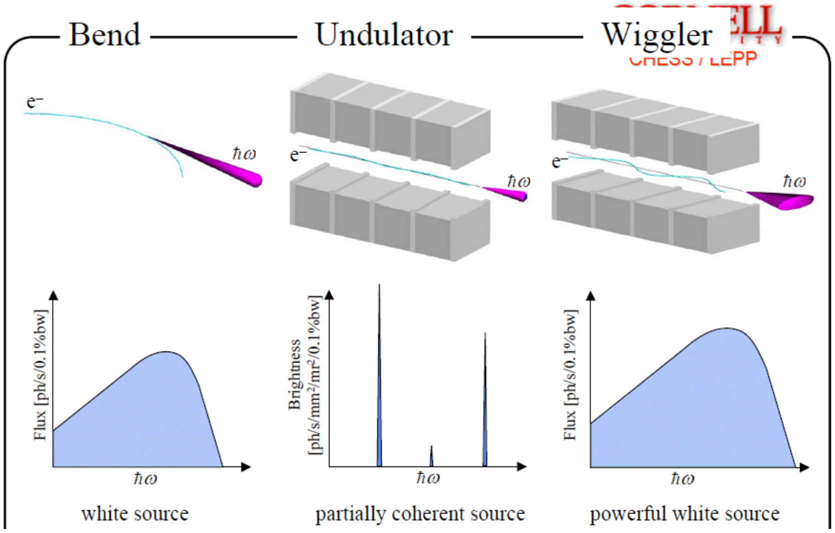
Periodic Magnets

- In rest frame of electron $\lambda^* = \dfrac{\lambda_U}{\gamma}$
- Electron oscillates coherently with (contracted) structure, and releases photons with the same wavelength.
- In the lab frame, this is Doppler shifted, so

$$\lambda = \frac{\lambda^*}{2\gamma} = \frac{\lambda_U}{2\gamma^2}$$

- So, λ on the order of 1cm ➔X-rays.

Bends, Undulators, and Wigglers*



*G. Krafft

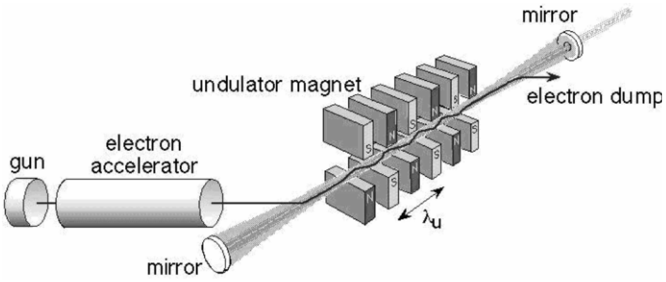
3^{rd} Generation (Undulator) Sources

- High Brightness
  - 10^{19} compared to 10^{16} for 2^{nd} generation sources
  - Emittance ~1-20 nm-rad
- A few Examples:
  - CLS
  - SPEAR-III
  - Soleil
  - Diamond
  - APS
  - PF
  - NSLS
  - BESSY
  - Doris
  - …

European Synchrotron Radiation Facility (ESRF)

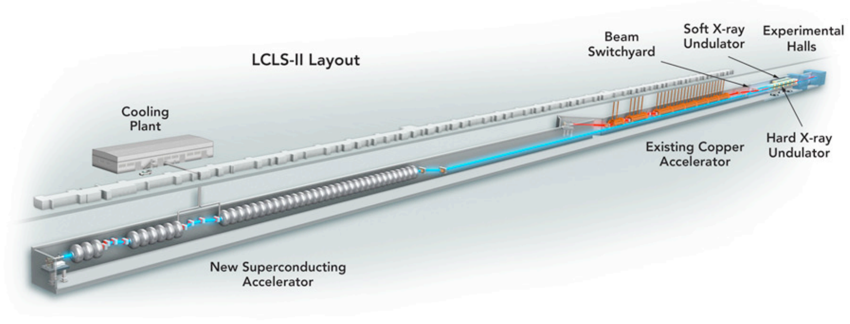
Fourth Generation

- Fourth Generation light sources generally utilize free electron lasers (FELs) to increase brightness by at least an order of magnitude over Third Generation light sources by using coherent production

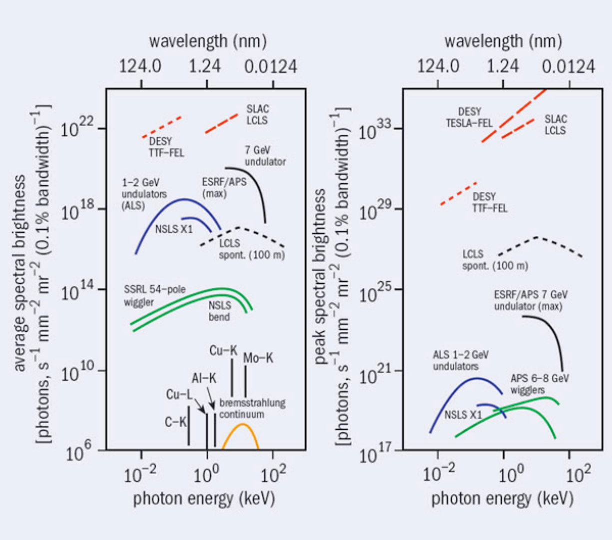


Next Big Thing in the US.

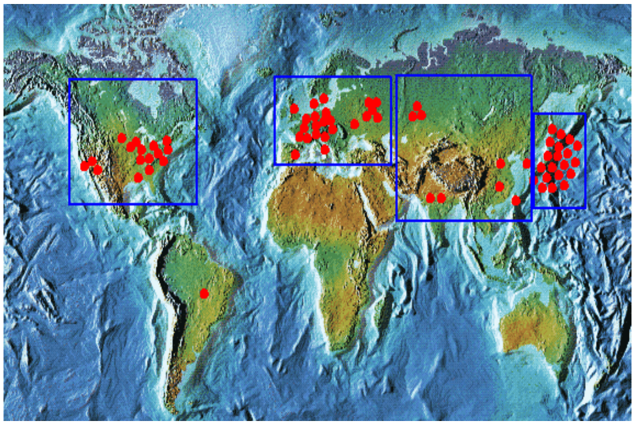
- LCLS-II at SLAC
  - 4 GeV superconducting linac
  - 1 MHz operation
  - X-rays up to 25 keV



Evolution of Parameters



Light Sources are a Huge (and growing) Industry



- Wikipedia lists about 60 light sources worldwide