MATCHING AND INSERTIONS

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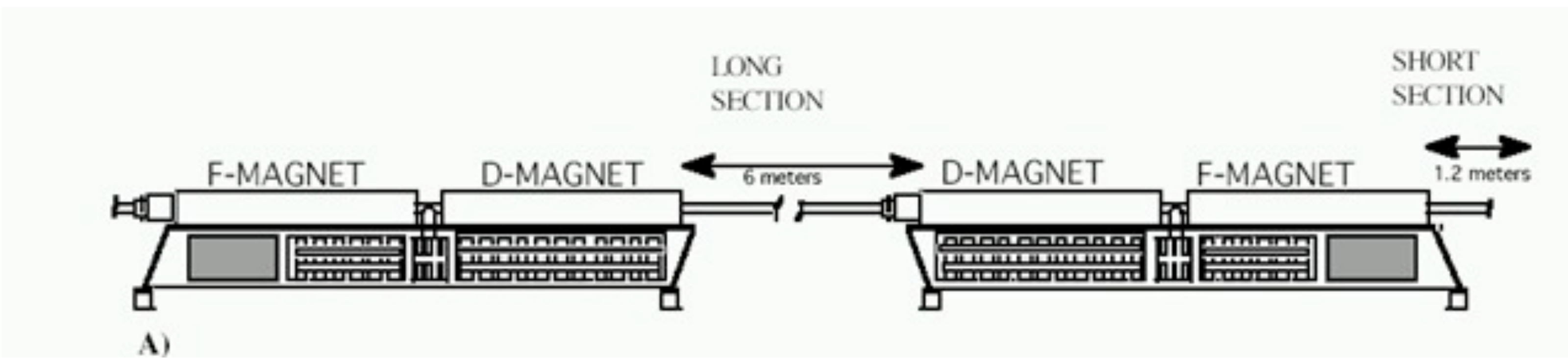
The Problem

- So far, we have talked about a synchrotron made out of identical FODO cells, with the space between the quads taken up by bend dipoles.
- The problem is that this is not particularly useful, because there's no place to put beam in or take it out, and no way to collide beams.
- One solutions is to design a "straight" into every cell. Example: the Fermilab Booster

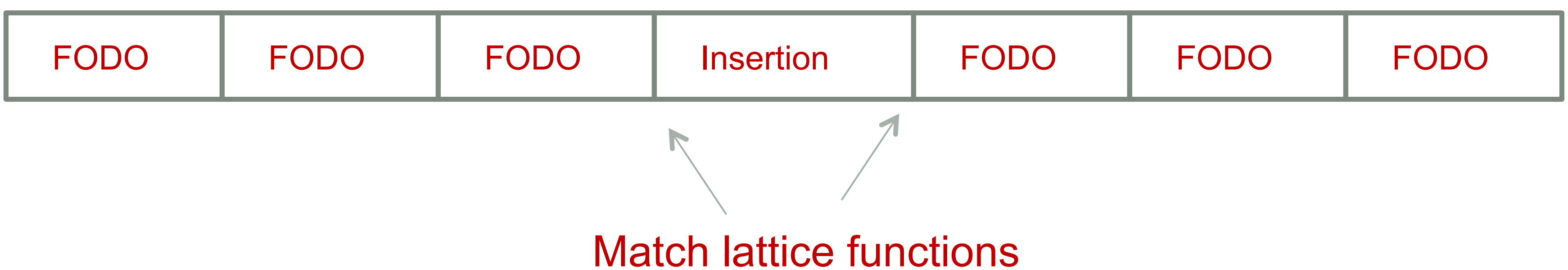


x24

- However, this is very wasteful of real estate. It would not be practical for the LHC.

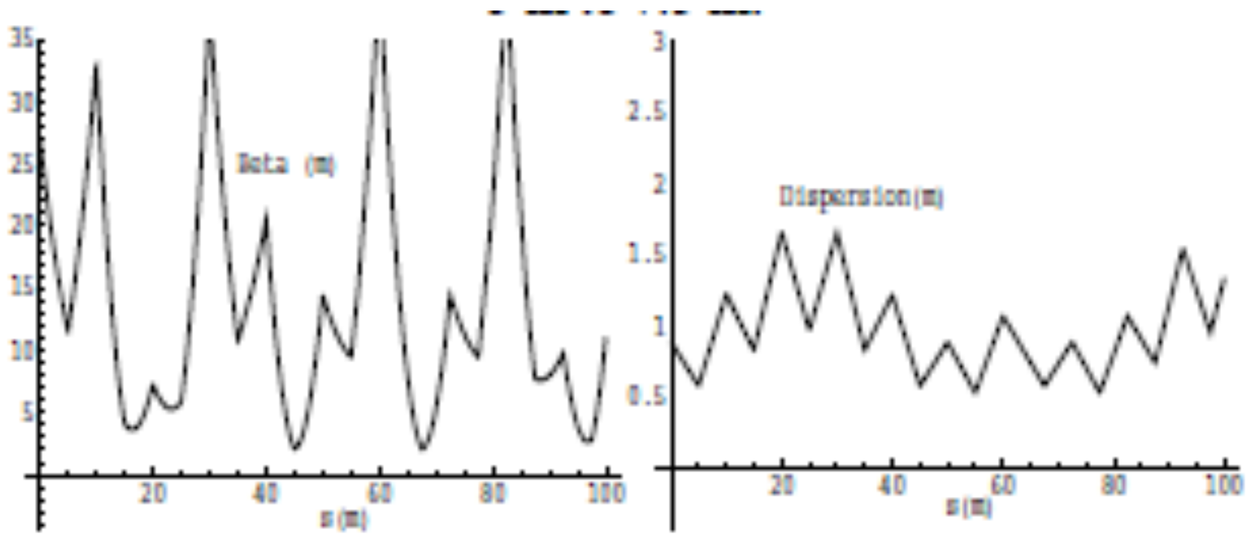
Insertions

- Since putting a empty straight section in every period is not practical, we need to explicitly accommodate the following in our design:
  - Locations for injection of extraction.
  - “Straight” sections for RF, instrumentation, etc
  - Low beta points for collisions
- Since we generally think of these as taking the place of things in our lattice, we call them “insertions”



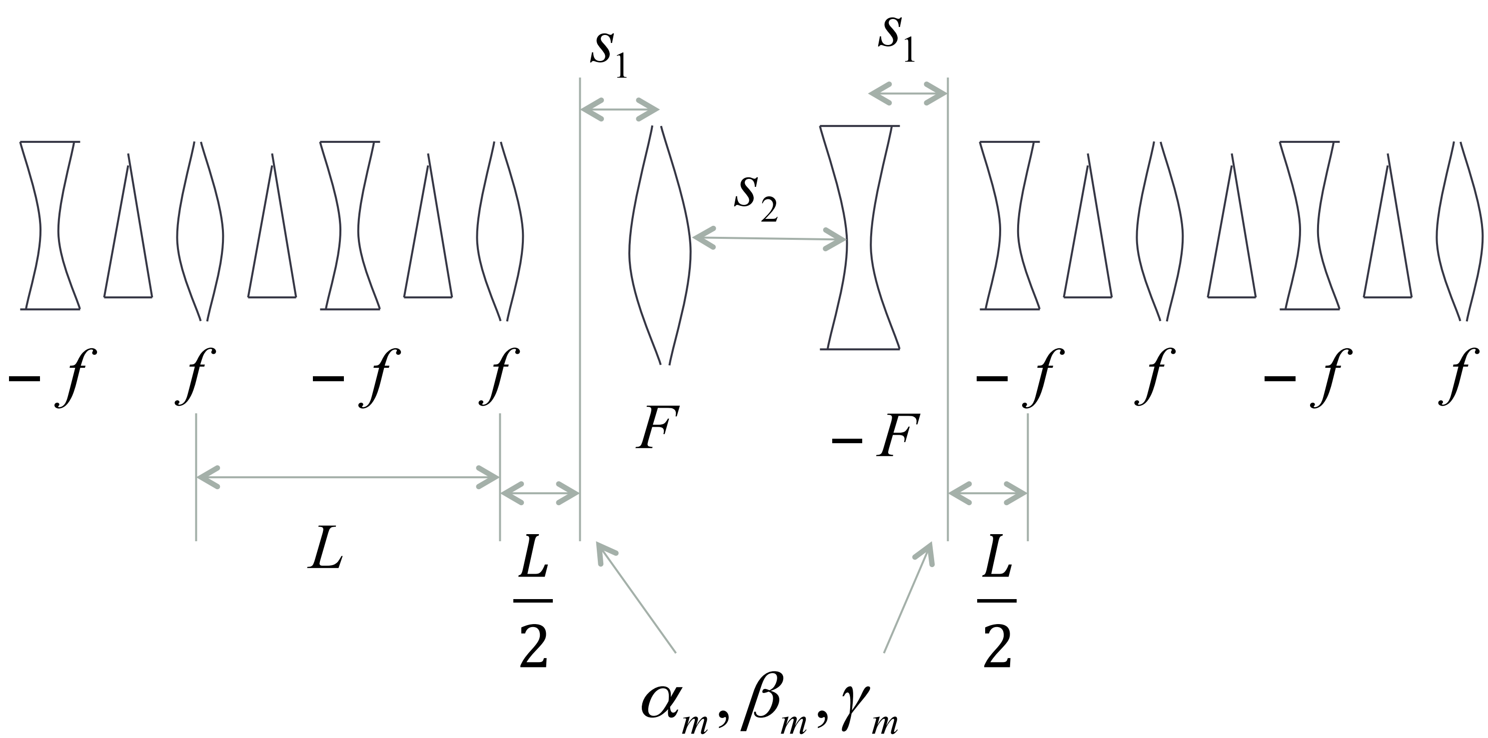
Mismatch and Beta Beating

- Simply modifying a section of the lattice without matching will result in a distortion of the lattice functions around the ring (sometimes called "beta" beating)
- Here's an example of increasing the drift space in one FODO cell from 5 to 7.5 m



Collins Insertion

- A Collins Insertion is a way of using two quads to put a straight section into a FODO lattice



- Where s_2 is the usable straight region

- Require that the lattice functions at both ends of the insertion match the regular lattice functions at those point

$$M=\begin{pmatrix}1 & s_1\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{1}{F} & 1\end{pmatrix}\begin{pmatrix}1 & s_2\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -\frac{1}{F} & 1\end{pmatrix}\begin{pmatrix}1 & s_1\\ 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}\cos\mu_I+\alpha_m\sin\mu_I & \beta_m\sin\mu_I\\ -\gamma_m\sin\mu_I & \cos\mu_I-\alpha_m\sin\mu_I\end{pmatrix}$$

Where μ_I is a free parameter

- After a bit of algebra

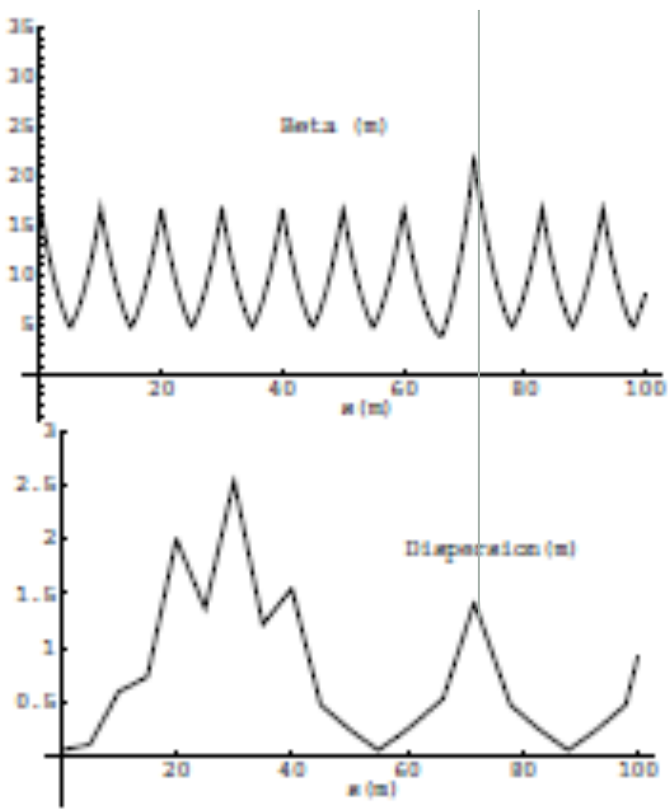
$$s_1=\frac{\tan\frac{\mu_I}{2}}{\gamma};\, s_2=\frac{\alpha^2\sin\mu_I}{\gamma};\, F=-\frac{\alpha}{\gamma}$$

- Maximize s_2 with $\mu_I=\pi/2$, α max (which is why we locate it *L/2* from quad)
- Works in both planes if $\alpha_x=-\alpha_y$ (true for simple FODO)

Dispersion Mismatch

- The problem with the Collins insertion is that it does *not* match dispersion, so just sticking it in the lattice will lead to distortions in the dispersion

Collins Insertion



Dispersion

- There is no way to bend a beam in a curved path without introducing dispersion.
- There are three cases dispersion is *desirable*:
  - To be used in combination with sextupoles to introduce chromaticity adjustment
  - To be used in combination with a collimation system to eliminate particles which are too far away from the nominal momentum.
  - Used in conjunction with various types of "cooling" to reduce energy spread.
- In all other cases, dispersion is a problem
  - Makes the beam bigger than it needs to be
  - Introduces problematic energy/position correlation

Dispersion Suppression

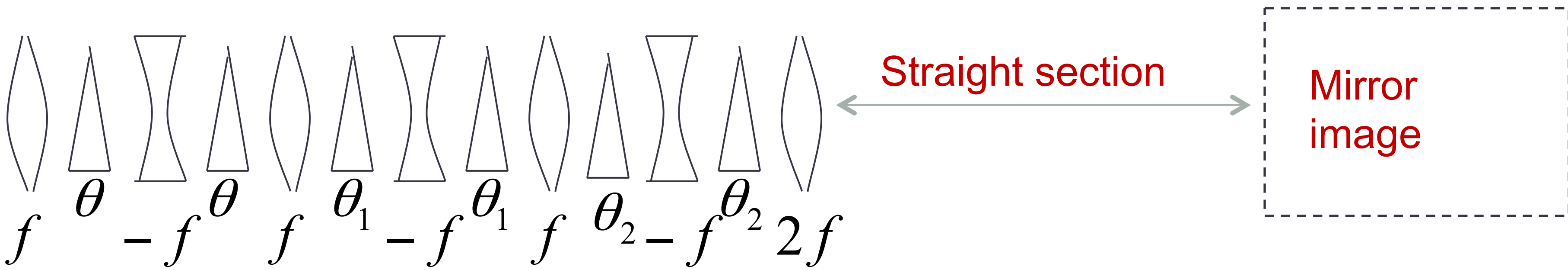
- Recall that dispersion propagates as

$$\begin{pmatrix} D_x(s) \\ D'_x(s) \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & d(s) \\ m_{21} & m_{22} & d'(s) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D_x(0) \\ D'_x(0) \\ 1 \end{pmatrix}$$

- For a straight section, *d(s)=d'(s)=0*, but dispersion will still propagate unless *D(0)=D'(0)=0* is also true.
  - ➔"Dispersion Suppression"

Dispersion Suppression (cont'd)

- On common technique is called the "missing magnet" scheme, in which the FODO cells on either side of the straight section are operated with two different bending dipoles and a half-strength quad



- Recall that the dispersion matrix for a FODO half cell is (lecture 4)

$$\mathbf{M}=\begin{pmatrix}1 & 1 & 0\\ -\frac{1}{2f} & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & L & \frac{L\theta}{2}\\ 0 & 1 & \theta\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 1 & 0\\ \frac{1}{f} & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & L & \frac{L\theta}{2}\\ 0 & 1 & \theta\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 1 & 0\\ -\frac{1}{2f} & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}1-\frac{L^2}{2f^2} & 2L\left(1+\frac{L}{2f}\right) & 2L\theta\left(1+\frac{L}{4f}\right)\\ -\frac{L}{2f^2}+\frac{L^2}{4f^3} & 1-\frac{L^2}{2f^2} & 2\theta\left(1-\frac{L}{4f}-\frac{L^2}{8f^2}\right)\\ 0 & 0 & 1\end{pmatrix}$$

- So we solve for

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \mathbf{M}(\theta = \theta_2)\mathbf{M}(\theta = \theta_1)\begin{pmatrix} D_m \\ D'_m \\ 1 \end{pmatrix}$$

- Where D_m and D'_m are the dispersion functions at the end of a normal cell (for a simple lattice, $D'_m=0$)
- We get the surprisingly simple result

$$\theta_1 = \theta\left(1 - \frac{1}{4\sin^2\frac{\mu}{2}}\right); \theta_2 = \theta\frac{1}{4\sin^2\frac{\mu}{2}}$$

"Missing Magnet" Cofiguration

- If we look at our solution

$$\theta_1 = \theta\left(1 - \frac{1}{4\sin^2\frac{\mu}{2}}\right); \theta_2 = \theta\frac{1}{4\sin^2\frac{\mu}{2}}$$

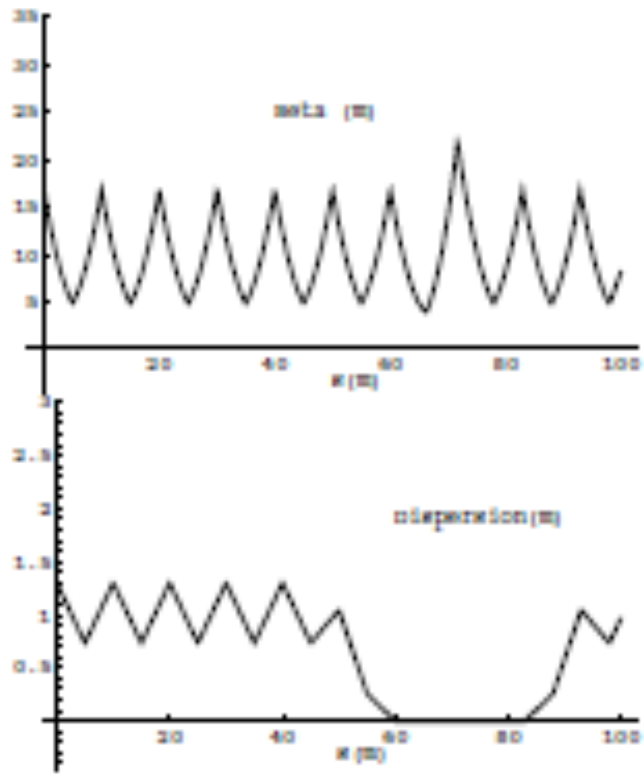
- And consider the case θ=60°, we get

$$\theta_1 = 0 \qquad \theta_2 = \theta$$

- So the cell next to the insertion is normal, and the next one has no magnets, hence the name "missing magnet".

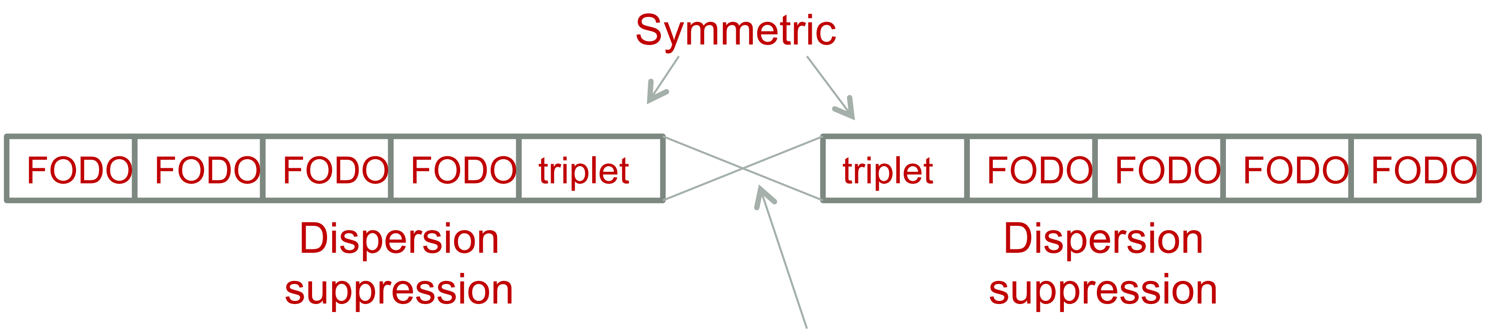
Combining Insertions

- Because the Collins Insertion has no bend magnets, it cannot generate dispersion if there is none there to begin with, so if we put a Collins Insertion inside of a dispersion suppressor, we match both dispersion and the lattice functions.



Low β Insertions

- In a collider, we will want to focus the beam in both planes as small as possible.
- This can be done with a symmetric pair of focusing triplets, matched to the lattice functions (dispersion suppression is assumed)



$$\beta = \beta^*; \alpha = 0$$

- Recall that in a drift, β evolves as

$$\beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2 = \beta^* + \frac{s^2}{\beta^*}$$

Where *s* is measured from the location of the waist

Phase Advance of a Low Beta Insertion

- We can calculate the phase advance of the insertion as

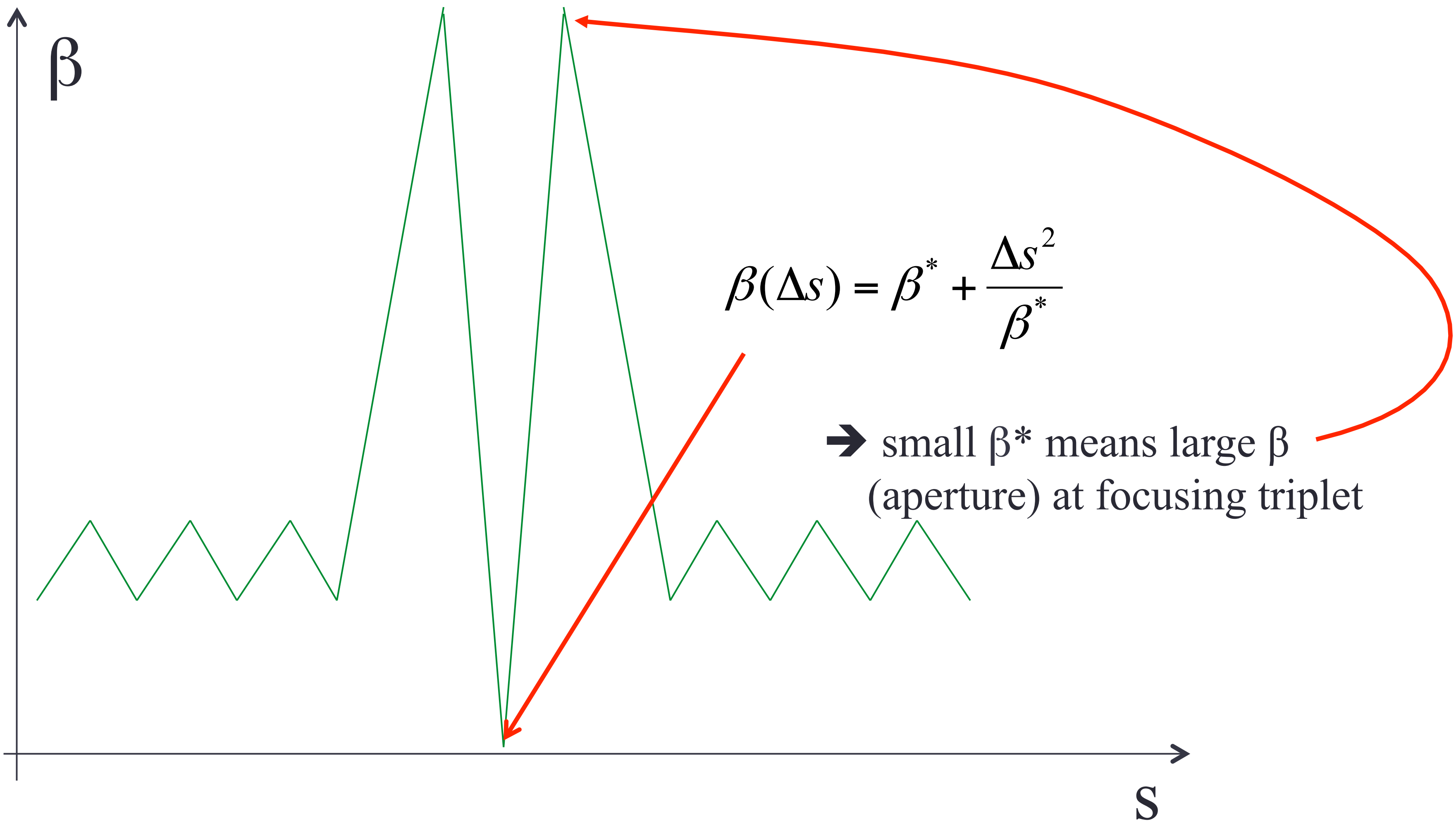
$$\Delta\psi = \int_{-L/2}^{L/2} \frac{ds}{\beta} = \frac{1}{\beta^*} \int_{-L/2}^{L/2} \frac{ds}{\left(1+\left(\frac{s}{\beta^*}\right)^2\right)} = 2\tan^{-1}\left(\frac{L}{2\beta^*}\right)$$

- For L>>β*, this is about π, so $\mathbf{M} = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix} \approx -\mathbf{I}$

$$\begin{pmatrix} \alpha(L/2) \\ \beta(L/2) \\ \gamma(L/2) \end{pmatrix} = \begin{pmatrix} (m_{11}m_{22}+m_{12}m_{21}) & (-m_{11}m_{21}) & (-m_{12}m_{22}) \\ (-2m_{11}m_{12}) & (m_{11}^2) & (m_{12}^2) \\ (-2m_{21}m_{22}) & (m_{21}^2) & (m_{22}^2) \end{pmatrix} \begin{pmatrix} \alpha(-L/2) \\ \beta(-L/2) \\ \gamma(-L/2) \end{pmatrix} \approx \mathbf{I} \begin{pmatrix} \alpha(-L/2) \\ \beta(-L/2) \\ \gamma(-L/2) \end{pmatrix}$$

- Matching guaranteed if insertion is symmetric!

Optics Near Low-β Insertion



$$\beta(\Delta s) = \beta^* + \frac{\Delta s^2}{\beta^*}$$

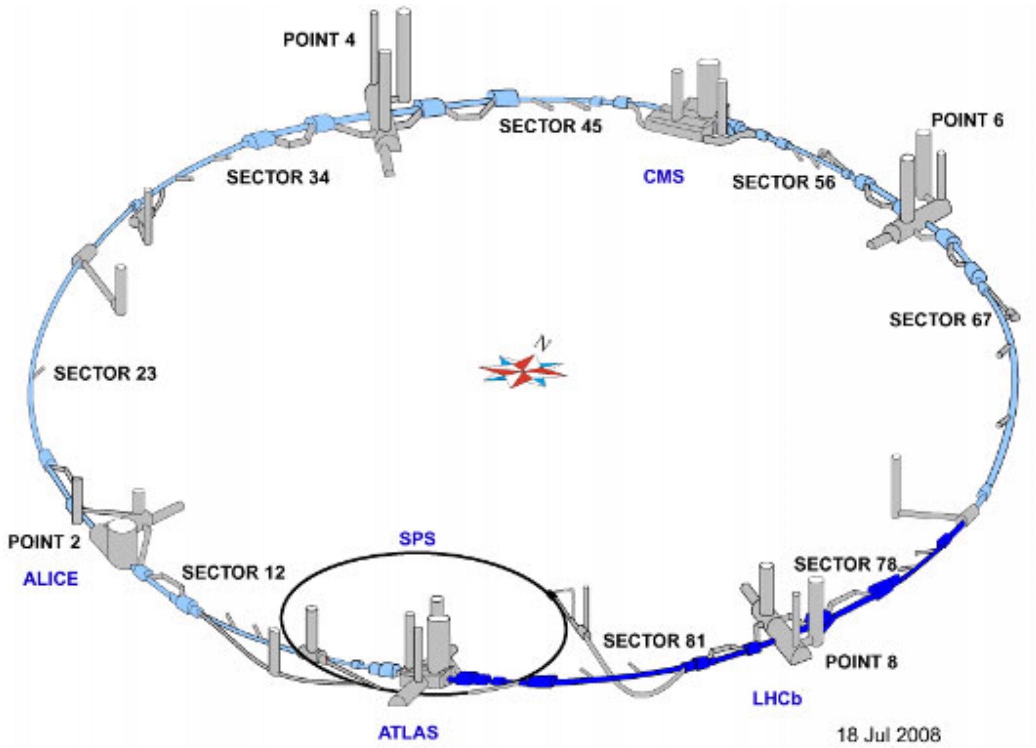
➔ small β* means large β (aperture) at focusing triplet

Putting the Pieces Together

- So now we see that in general, a synchrotron will contain
  - A series of identical FODO cells in most of the ring.
  - Straight sections, with modified cells on either end.
  - Dispersion suppression before and after these straight sections
- If it's a collider, it will also contain
  - One or more low beta insertions with dispersion suppression on either side.
  - The beta function will be very large on either side of the low beta point

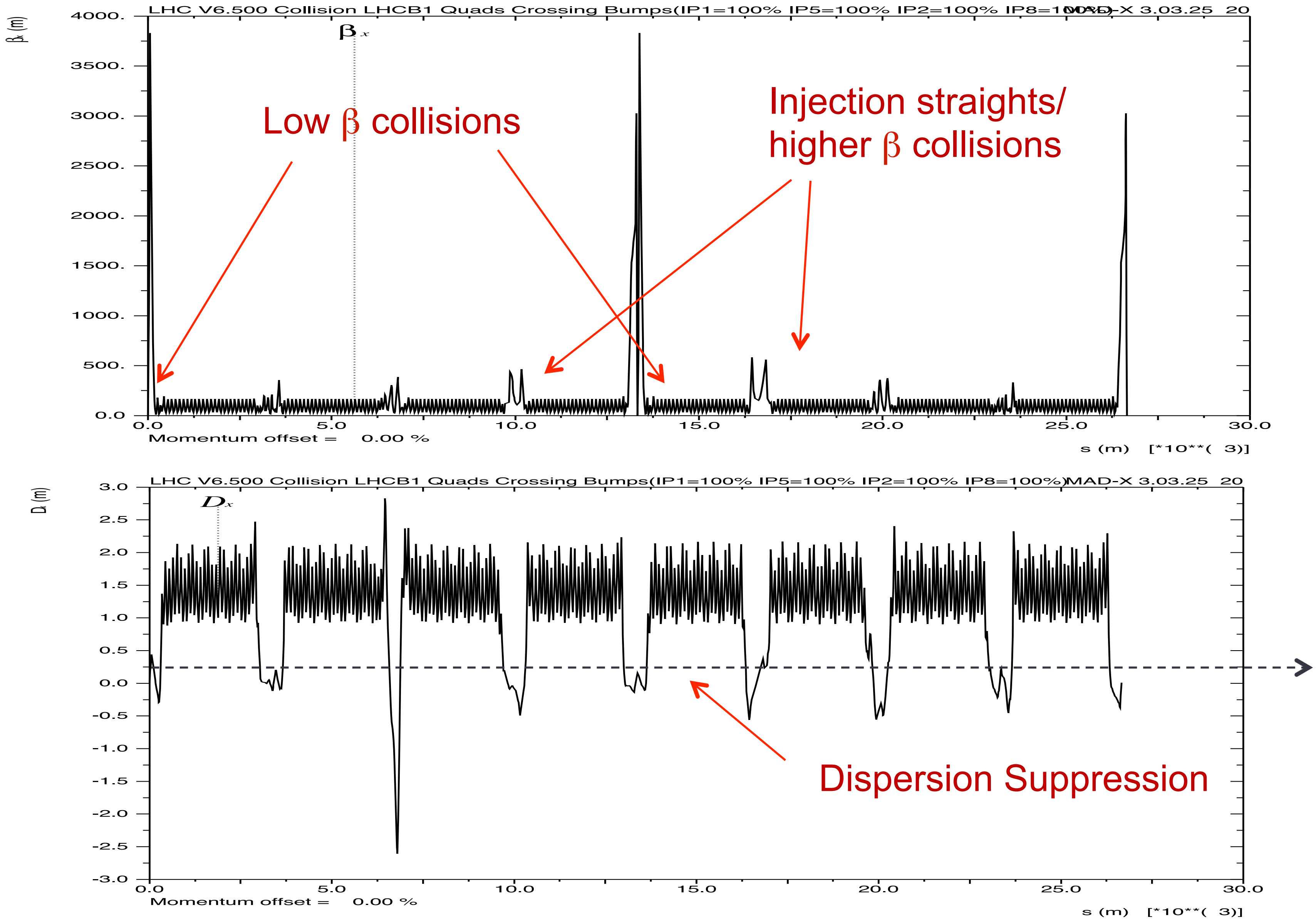
Example: LHC

- Recall the LHC layout



- Superperiodicity of 8
- Need insertions for two low beta collision regions (ATLAS, CMS)
- Two higher beta collision regions (ALICE, LHCb), which double as injections points.
- Other straights for RF cavities, beam extraction, etc.

LHC Optics (out of date)

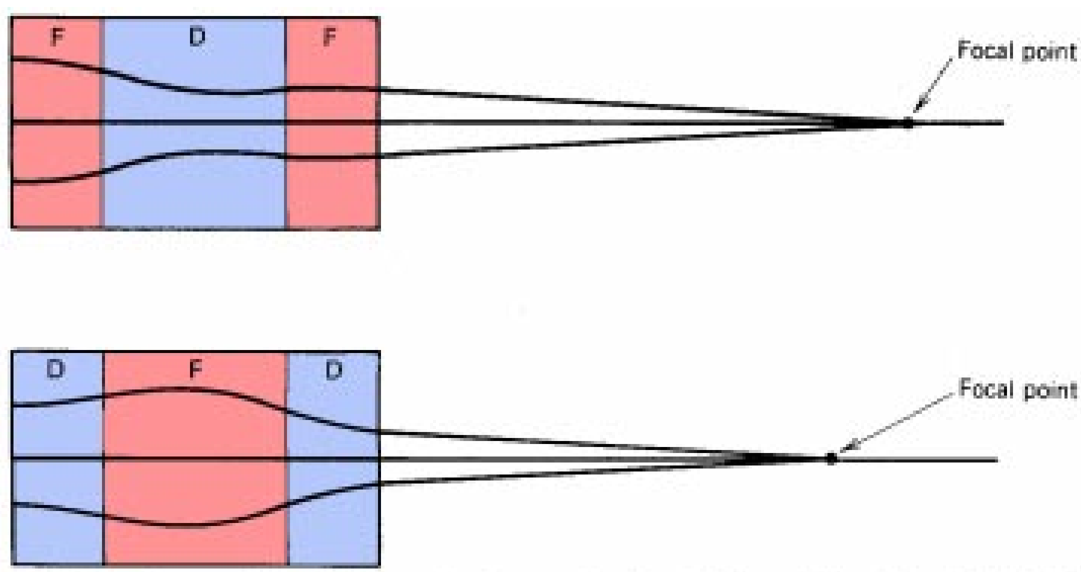


Beam Line Issues

- Beam lines are typically built in discrete sections:
  - Matching (to a source, injection point, or extraction point)
  - Transport:
    - The FODO cells we've been talking about
  - Bends
    - Designed as "achromats" to suppress dispersion!
  - Focus (or "waist")
    - Uses quad triplet to minimize beta in both planes
  - Collimation sections
    - 90°apart in phase space to clean up 2D phase space

Final Focus Triplet

- As we saw, our normal FODO cell has maxima in one plane where the minima are in the other.
- For targets or collisions, we want small beta functions in both planes.
- This optical problem can be solved with a triplet
  - Middle quad ~twice the strength of outer quads (HW problem for next week)



Dispersion Suppression

- Any bend section will introduce dispersion. After the bend, it will propagate as

$$\begin{pmatrix} D_x(s) \\ D'_x(s) \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D_x(0) \\ D'_x(0) \\ 1 \end{pmatrix}$$

- It will never go away unless we explicitly suppress it in the design

Dispersion due to a Dipole

- We already solved for the dispersion introduced by a bend dipole

$$\begin{pmatrix} D_{out} \\ D'_{out} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & L & \frac{L\theta}{2} \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D_{in} \\ D'_{in} \\ 1 \end{pmatrix}$$

- So if the beam line has no dispersion going into the dipole, it will exit with dispersion

$$D = \frac{1}{2} L\theta$$
$$D' = \theta$$

Propagation of Dispersion

- In the absence of additional bends, the beam the dispersion will propagate just like any other orbit.

$$\begin{pmatrix} D(s) \\ D'(s) \\ 1 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{\beta(s)}{\beta_0}}\left(\cos\Delta\psi + \alpha_0 \sin\Delta\psi\right) & \sqrt{\beta_0\beta(s)}\sin\Delta\psi & 0 \\ \frac{1}{\sqrt{\beta_0\beta(s)}}\left(\left(\alpha_0 - \alpha(s)\right)\cos\Delta\psi - \left(1+\alpha_0\alpha(s)\right)\sin\Delta\psi\right) & \sqrt{\frac{\beta_0}{\beta(s)}}\left(\cos\Delta\psi - \alpha(s)\sin\Delta\psi\right) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2}L\theta \\ \theta \\ 1 \end{pmatrix}$$

- We'll consider the special case of $\Delta\psi = n\pi$
- If, in addition, we make the line symmetric, so the lattice functions are the same at the end as the beginning, this simply reduces to

$$\begin{pmatrix} D(s) \\ D'(s) \\ 1 \end{pmatrix} = \begin{pmatrix} (-1)^n & 0 & 0 \\ 0 & (-1)^n & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2}L\theta \\ \theta \\ 1 \end{pmatrix} = (-1)^n \begin{pmatrix} \frac{1}{2}L\theta \\ \theta \\ 1 \end{pmatrix}$$

Canceling Dispersion

- If we put a second magnet at the end of this line, it will modify the dispersion as

$$\begin{pmatrix} D_{out} \\ D'_{out} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{L\theta_2}{2} \\ 0 & 1 & \theta_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} (-1)^n \frac{L\theta}{2} \\ (-1)^n \theta \\ 1 \end{pmatrix}$$

- So we can cancel the dispersion by setting

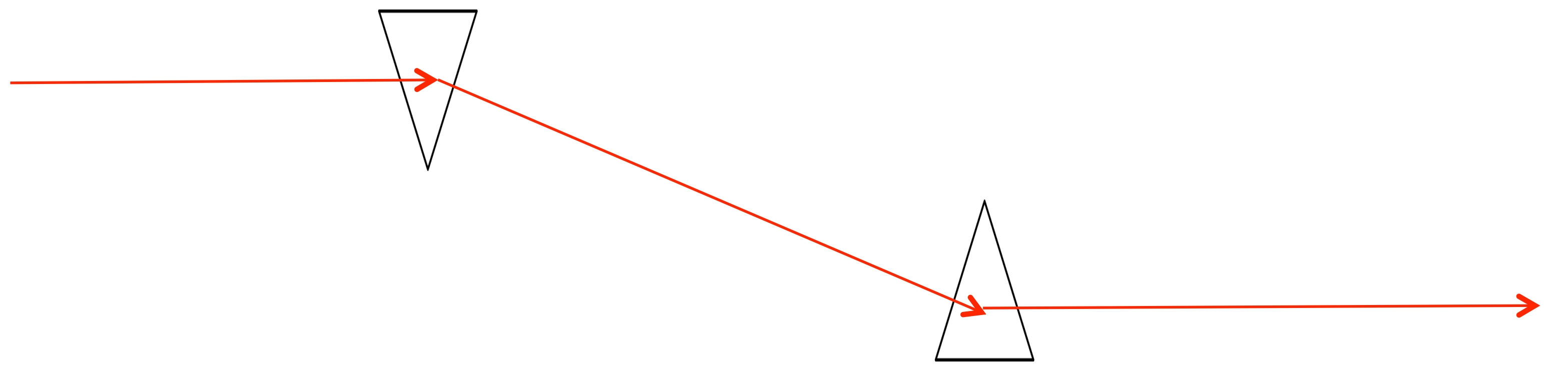
$$\theta_2 = (-1)^{n-1} \theta$$

and the dispersion will remain zero until the next bend magnet

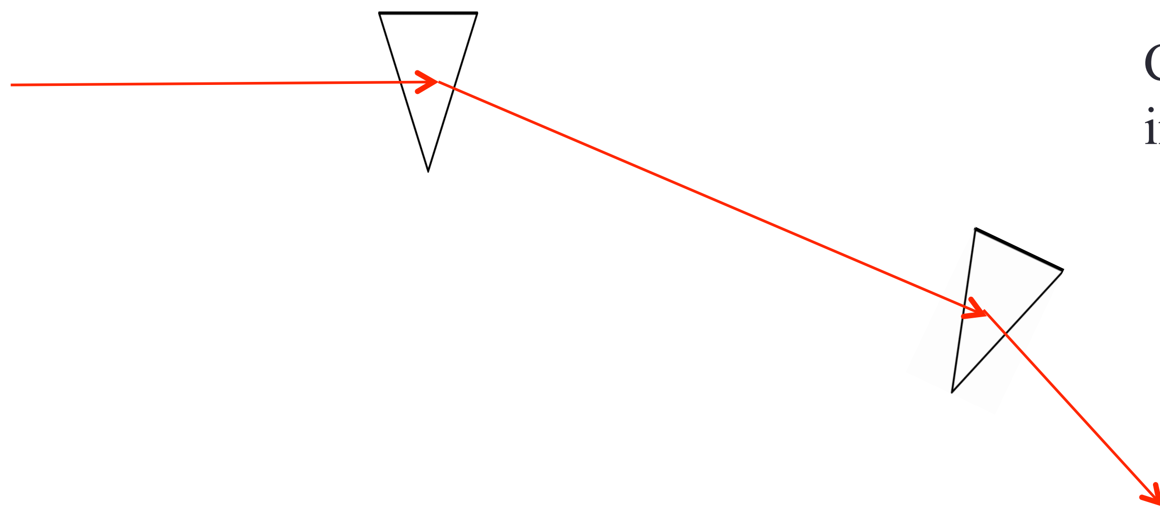
- Such a section of beam line is referred to as an "achromat"

Achromats

- If the line has 360° of phase advance, we can cancel dispersion with an opposite sign dipole➔"dogleg achromat"



- If the line has 180°of phase advance, we can cancel dispersion with a same sign dipole ➔"double-bend achromat"



Can be incorporated into rings

Example: Mu2e Proton Beam

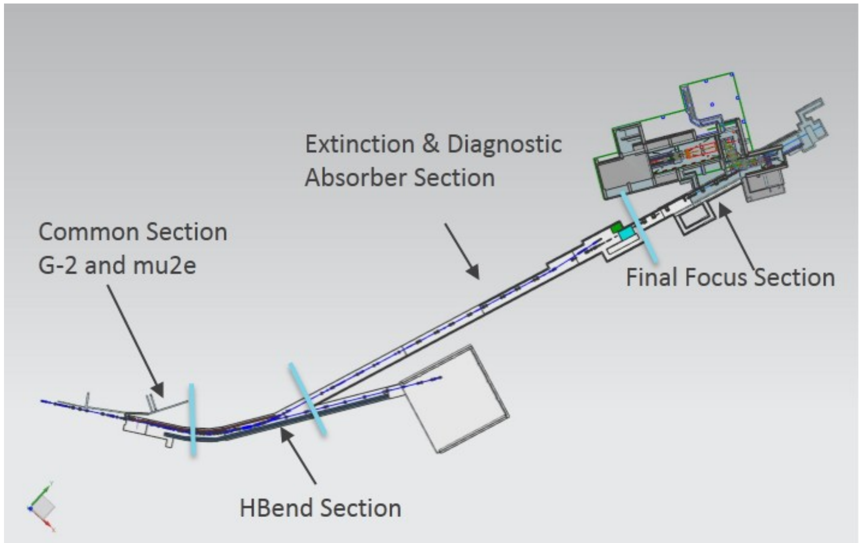
Muon production target

triplet

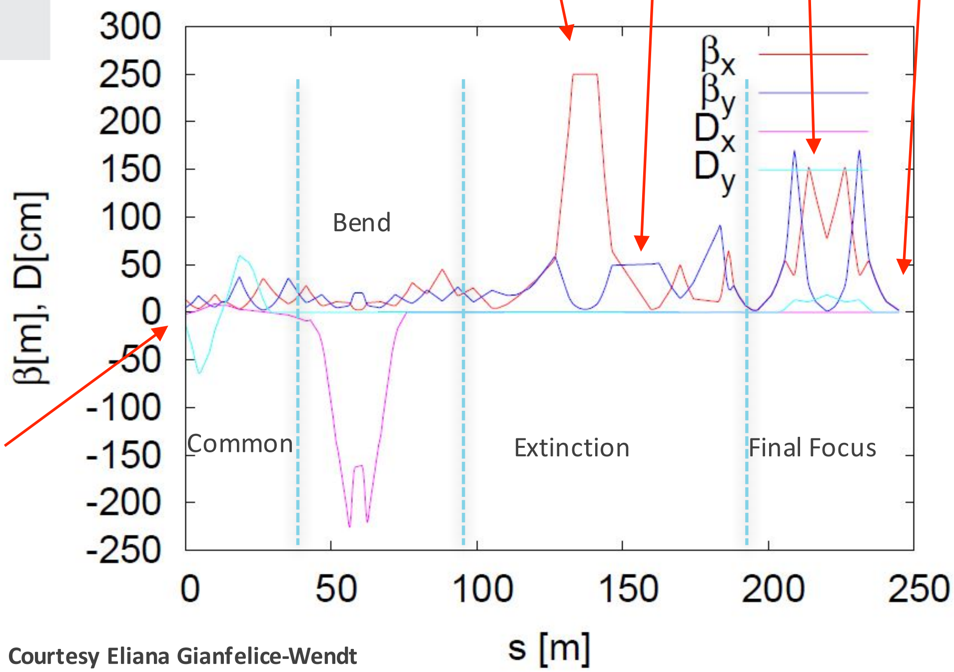
collimator

weird magnet requirement

"achromat"



matching



Courtesy Eliana Gianfelice-Wendt