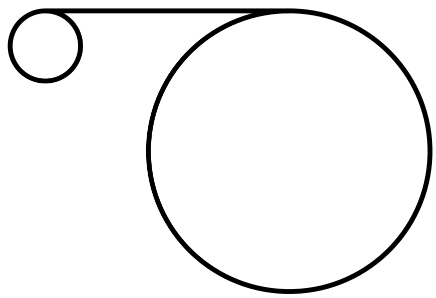
Accelerator Fundamentals Homework 6

1. Starting with our go-to synchrotron: 3 km in circumference, protons injected with a kinetic energy of $K = 10$ GeV, with an RF system of harmonic h=588, 1 MV total RF voltage, and transition gamma γ_t= 25. We now assume that beam is injected into this synchrotron from a lower energy synchrotron that has exactly 1/6th the circumference, a maximum kinetic energy of K=10 GeV, and a transition gamma of γ_t=5.

$C = 500$ m
$K_{min} = 1$ GeV
$K_{max} = 10$ GeV
$\gamma_t = 5$
h=?
$V_{RF} = ?$



$C = 3$ km
$K_{min} = 10$ GeV
$K_{max} = 100$ GeV
$\gamma_t = 25$
h=588
$V_{RF} = 1$ MV

When the first synchrotron reaches K=10 GeV, it stops accelerating and the beam is transferred to the second synchrotron, which then begins to accelerate (i.e, the high energy synchrotron is not accelerating when the beam is injected. Using your answers from the last two homework sets, answer the following questions about the low energy synchrotron at its maximum energy, after it has stopped accelerating, just before the beam is transferred to the high energy synchrotron:

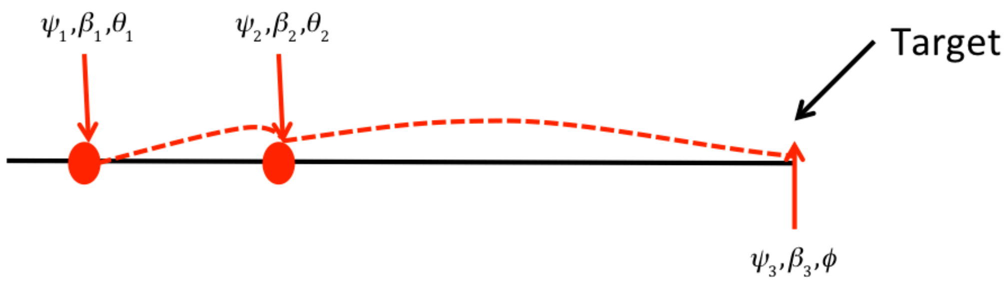
   a. What is the slip factor η?
   b. If the RF system has the same frequency as the high-energy synchrotron, what is the harmonic number *h*?
   c. What RF voltage V_{RF} will be required to correctly match the beam bunches to the buckets in the high-energy synchrotron in the transfer?
   d. Answer questions (b) and (c) assuming the RF in the low energy synchrotron is running at *half* the frequency of the high energy synchrotron, and transferring into every other bucket.

2. [Based on a question I got after class] In class ("Off-momentum Particles" lecture, p. 9-10), we wrote the full transfer matrix of a bend dipole, including dispersion, as

$$\begin{pmatrix} 1 & L & \frac{L\theta}{2} \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

In other words, we treated the 2x2 part of the matrix as a simple drift $\left(x''=0\right)$, but that's not quite correct, is it? We derived in class that while going through the dipole, the curved coordinate system will lead to an effective "centripetal force" focusing term, given by ("Transverse motion" lecture) $x''+\left(\frac{1}{\rho^2}\right)x=0$. Use this to write the exact 2x2 transfer matrix (i.e. the "piecewise solution") for the bend dipole, and show that in the limit $L \ll \rho$, it reduces to $M \approx \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$, as we assumed.

3. When focusing a beam on a target, it is useful to be able to use corrector magnets to *independently* correct the position and angle at the target. Let's just worry about the angle for the moment. Assuming we have two correctors upstream of the beam target, with lattice parameters β_1,ψ_1 and β_2,ψ_2 , respectively, what angles θ_1 and θ_2 would I set them to keep the position at the target fixed at 0, but have it arrive with an angle of ϕ, if the lattice parameters at the target are β_3,ψ_3 [1]



[1] Hint: this drops very quickly out of the three-bump equation we derived in class.