Accelerator Fundamentals Homework 5

1. [This is a follow-up to problem 3 from HW#4] We have a synchrotron that is 3 km in circumference. This synchrotron is initially not accelerating, and we inject protons at K=10 GeV. After they are injected, they are accelerated from 10 to 100 GeV in 1 second, after which they stop accelerating prior to extraction. The RF system has a harmonic of 588, and a total voltage of 1MV. The transition gamma (γ_t) is 25. You may treat the period as approximately constant (v=c) for this problem. Last time, you calculated the following
   a. What is the period of the synchrotron [s]? 10 μs
   b. What is the frequency of the RF system [Hz]? 58.8 MHz
   c. What is the slip factor η at the injection energy? -.0058
   d. What is the slip factor η at the extraction energy? +.0015
   e. What is the synchronous phase angle ϕ_s [degrees] at injection, before the beam starts to accelerate? 0°
   f. What is the synchronous phase angle ϕ_s [degrees] just as beam begins to accelerate? 64.2°
   g. What is the synchronous phase angle ϕ_s [degrees] just before beam stops accelerating? 115.8°

   Now answer the following questions about the same accelerator

   h. What is the synchrotron tune ν_s at those three points?
   i. What is the longitudinal beta function β_L [s/eV] at each of the points?
   j. What is the bucket height ΔE_b [eV] at injection (i.e. before the beam starts accelerating)?
   k. If the injected beam has an RMS energy spread of $\sigma_E = \frac{1}{5}\Delta E_b$, what will be the longitudinal RMS emittance ϵ_L [eV-s] at injection?
   l. If the bunch is matched, what is σ_t [s] at this point?
   m. If the bunch remains matched, what are σ_E and σ_t at the other two points (just after acceleration starts and just before it stops)?

2. All of our calculations assume that the nominal beam trajectory passes through the center of each quad, and it's very important to align the quads so that this happens. If a quad is out of alignment, the particles will see a net dipole term, which will distort the orbit. Using all the parameters that you have calculated for the LHC at maximum energy (7000 GeV), answer the following questions:

a. If one of the focusing quadrupoles is misaligned by 1 mm in *x*, what is the integrated dipole field [T-m] that particles will see? (just worry about the magnitude, not the sign.).
b. How big of an angular deflection θ will this cause to the beam?
c. Based on our formula for closed orbit distortion ("Closed Orbit Distortion" lecture), what is the maximum deviation that this misalignment will cause in the ring? The LHC tune in the horizontal plane is 64.3. (hint: you don't have to worry about specific phase advances. You can assume that at some points in the ring, the cosine term will be very close -1 or +1).

3. I have a synchrotron consisting of N_{cell} identical FODO cells, each with a maximum and minimum beta of β_{max} and β_{min}, respectively, and all quads have a nominal focal length $\pm\frac{1}{f_0}$. What will be the change to the tune of the machine if I change ALL the quad focal lengths by a fraction amount $\frac{\Delta f}{f_0}$ $\left(\Delta f \ll \mathrm{f}_0\right)$, say by changing the current in the bus[1]?

[1] Hint: approximate the change to the focal length by an extra quadrupole at every point, and just plug the value into the tune shift formula. If you find yourself doing a lot of work, you're not doing the problem right.