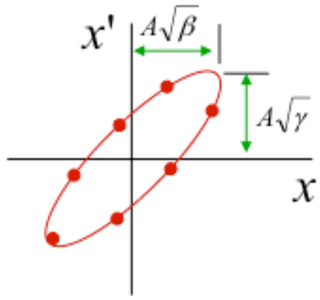
Accelerator Fundamentals Homework 3

1. Assume we have a very strange distribution of particles; namely, a bunch of particles matched to the local lattice functions, all with the *same* amplitude *A*, but random phase angles ψ_i. As we have shown, these will populate an ellipse as shown below.



$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = A^2$$
$$\beta\gamma - \alpha^2 = 1$$

We have also shown (Transverse Motion) that in this case, the position and angle of each particle *i* will be given by

$$x_i = A\sqrt{\beta_x}\cos(\psi_i + \delta)$$
$$x'_i = -A\frac{1}{\sqrt{\beta_x}}\left(\alpha\cos(\psi_i + \delta) + \sin(\psi_i + \delta)\right)$$

where ψ_i is randomly distributed. (You could also think of this as a single particle returning over and over again to the same point). If I measure the positions and angles of the particles:

a. Show that the RMS value for the x distribution will be

$$\sigma_x \equiv \sqrt{\frac{1}{N}\sum x_i^2} = \sqrt{\frac{A^2\beta_x}{2}}$$

b. Based on our usual relationship between the RMS and the emittance, write an expression for the unnormalized "emittance" ϵ of this distribution in terms of *A*. (warning: it will not make much sense in terms of the picture).

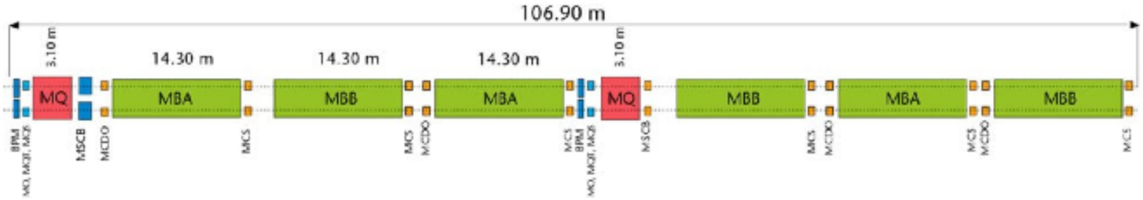
c. Using *this* expression for ϵ, prove the following for this distribution of particles

$$\sigma_{x'} \equiv \sqrt{\frac{1}{N}\sum x'^2_i} = \sqrt{\gamma_x\epsilon}$$
$$\sigma_{xx'} \equiv \frac{1}{N}\sum x_i x'_i = -\epsilon\alpha_x$$

d. Use these relationships to write an expression for ϵ in terms of only the measured distributions σ_x, $\sigma_{x'}$, and $\sigma_{xx'}$.

e. Now that you've calculated ϵ, write the expressions for the lattice functions β, α, and γ at this point in terms of ϵ, σ_x, $\sigma_{x'}$, and $\sigma_{xx'}$. (Hint: this is just trivially reversing the steps of (b) and (c). This relationship is true in general. It's just harder to prove the general case)

2. The LHC is has a design kinetic energy of 7000 GeV. The basic LHC FODO cell is shown below



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It has a full length of 106.9 m ($L = 53.45\ \mathrm{m}$), and a phase advance per cell of $\mu = 90^\circ$. Using the formulas we derived in class (Transverse Motion), calculate the following:

a. The focal length *f* of the quadrupoles required to give the specified phase advance [m].
b. If the quadrupoles are 3.1 m long, what gradient B' is required to achieve this focal length at full beam energy [T/m]
c. β_{max} at the center of the focusing quads [m].
d. $\beta_{\min}$ at the center of the defocusing quads [m].
e. The normalized RMS emittance of the beam is 2.75 μm. What is the physical transverse RMS of the beam (σ) in the middle of the focusing quads at the injection energy of 400 GeV [mm]?
f. What is the transverse RMS of the beam at the same location at the maximum energy [mm]?

3. Use the MADX simulation program to calculate the lattice parameters for the LHC FODO cell described in problem 2. You may run it on one of the lab computers, or download it to your own computer from http://cern.ch/madx .

You can start with the MADX script I showed in class (see notes Transverse Motion). You may download the script that was shown in class and modify it for your needs. It is located on the website (http://home.fnal.gov/~prebys/misc/uspas_2018/) under the “Homework” section. You will need to change the length and strength of the quadrupoles, as well as the length of the drifts to match the LHC. Assuming you change the name of the file to “lhc.madx”, the program is run by typing “madx < lhc.madx”[1]. It will generate two files: a PostScript graphics file with the lattice functions plotted, and a text file, with the lattice parameters at various points. Note that the lattice values are given at the *end* of the element listed, so for example, you would find the lattice values for the middle of a quad listed on the line for the first half.

Compare the values for β_{max} and β_{min} that are calculated by MADX to the ones you calculated in problem 2.

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[1] You may have to explicitly specify the path. For example, on the lab computers, you will have to type: “\Program Files (x86)\mad\madx\madx.exe” < lhc.madx (with the quotation marks).