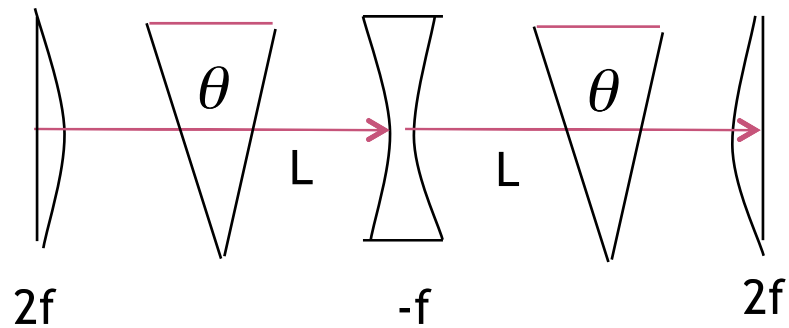
Accelerator Fundamentals Homework 5

1. Starting for the standard FODO cell with bend sectors that we discussed in class

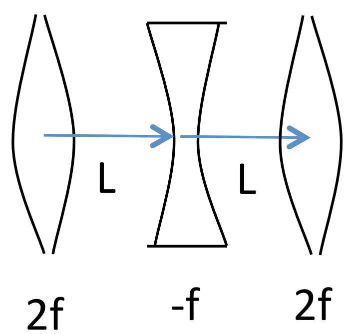


   Show that the dispersion is given by

$$D_{F,D} = \frac{\theta L\left(1 \pm \frac{1}{2}\sin\frac{\mu}{2}\right)}{\sin^2\frac{\mu}{2}}$$

   (hint: follow the discussion on p. 17-19 of Lecture 04 and use the symmetry to make a simplifying assumption about *D'*)

2. Consider a magnetic triplet formed by three quadrupoles, in which the center quadrupole has twice the strength of the outer two, and the magnets are separated by a distance much less than their focal lengths (L<<f).



   In this limit, write an expression for the effective focal length of the system, and show that it is the *same* in both planes. Draw a simple figure in each plane explaining qualitatively why this is so (hint: start with a beam parallel to the axis and treat it like a classical optics problem).

3. We have a synchrotron that accelerates protons from a kinetic energy of 10 to 100 GeV in 1 second, with the energy changing linearly with time. The circumference of the ring is 3km. The transition γ_T is 25.
   a. Calculate the revolution periods at
      i. Injection (minimum) energy.
      ii. Just below transition (say, γ =24).
      iii. Extraction (maximum) energy.

b. The RF system has a harmonic number of 588; that is, one revolution period corresponds to 588 RF cycles. Calculate the RF frequency in MHz at each of these three energies (this difference may seem small, but changing the resonant frequency of a cavity is always hard).
c. Calculate the slip factor (including sign) at each of the three energies.
d. If I have an RF voltage of 1 MV, what synchronous phase ϕ_s do I need at these three energies (hint: what is the change in energy per turn)?
e. Calculate the longitudinal beta β_L at each of the three energies.
f. Calculate the maximum bucket height ΔE_b at injection (hint: Longitudinal Motion lecture, p. 13, and remember the beam is not accelerating yet.)
g. Assuming the injected bunch is matched to the bucket with $\sigma_E = \frac{1}{4} E_b$, calculate σ_t and the (Gaussian) longitudinal emittance $\epsilon_L = \sigma_E \sigma_t$ at injection.
h. Assuming that longitudinal emittance is conserved, calculate σ_E and σ_t at each of the other two energies (hint: this is really easy based on the previous two answers).