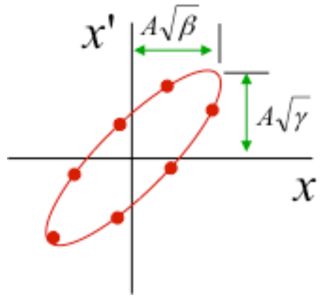
# Accelerator Fundamentals Homework 4 (Corrected!)

1. Assume we have a very strange distribution of particles; namely, a bunch of particles matched to the local lattice functions, all with the *same* amplitude *A*, but random phase angles $\psi_i$. As we have shown, these will populate an ellipse as shown below.




$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = A^2$$
$$\beta\gamma - \alpha^2 = 1$$

We have also shown (Transverse Motion p. 29) that in this case, the position and angle of each particle *i* will be given by

$$x_i = A\sqrt{\beta_x}\cos(\psi_i + \delta)$$
$$x'_i = -A\frac{1}{\sqrt{\beta_x}}\left(\alpha\cos(\psi_i + \delta) + \sin(\psi_i + \delta)\right)$$

where $\psi_i$ is randomly distributed. (You could also think of this as a single particle returning over and over again to the same point). If I measure the positions and angles of the particles:

a. Show that the RMS value for the x distribution will be

$$\sigma_x \equiv \sqrt{\frac{1}{N}\sum x_i^2} = \sqrt{\frac{A^2\beta_x}{2}}$$

b. Based on our usual relationship between the RMS and the emittance, write an expression for the unnormalized "emittance" $\epsilon$ of this distribution in terms of *A*. (warning: it will not make much sense in terms of the picture).

c. Using *this* expression for $\epsilon$, prove the following for this distribution of particles

$$\sigma_{x'} \equiv \sqrt{\frac{1}{N}\sum x'^2_i} = \sqrt{\gamma_x\epsilon}$$
$$\sigma_{xx'} \equiv \frac{1}{N}\sum x_i x'_i = -\epsilon\alpha_x$$

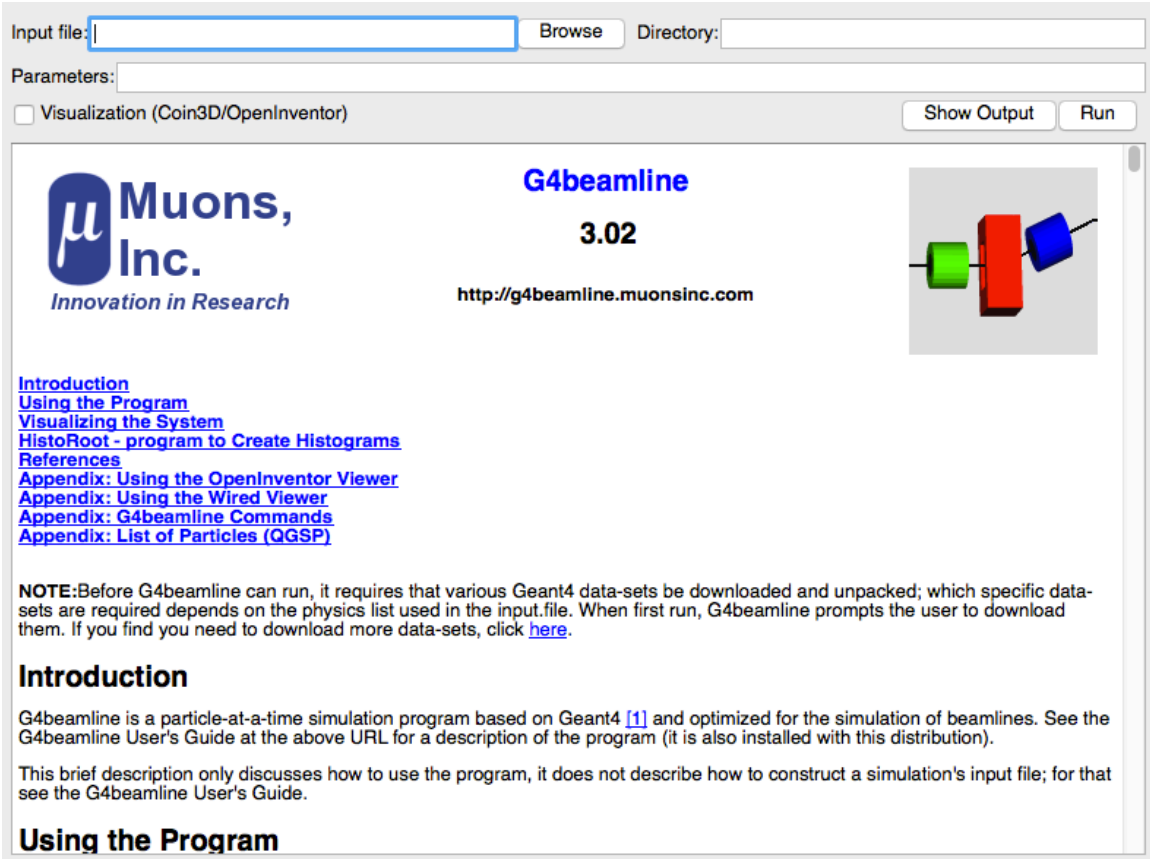
d. Use these relationships to write an expression for $\epsilon$ in terms of only the measured distributions $\sigma_x$, $\sigma_{x'}$, and $\sigma_{xx'}$.

e. Now that you've calculated $\epsilon$, write the expressions for the lattice functions $\beta$, $\alpha$, and $\gamma$ at this point in terms of $\epsilon$, $\sigma_x$, $\sigma_{x'}$, and $\sigma_{xx'}$. (Hint: this is just trivially reversing the steps of (b) and (c), except that now you're *starting* with an emittance that you calculated based only on the distributions, so you've gotten all your numbers without knowing anything *a priori* about the lattice).

Note: all of these relationships for the lattice functions and unnormalized RMS emittances in terms of RMS distributions are true in general for any real particle distribution, as long as it's matched. It's just a little harder to prove the general case.

2. For this problem, we will revisit the LHC FODO cell from yesterday's homework. This time we'll use the g4beamline simulation program to run the simulation and the HistRoot program to analyze the data. These programs are installed on the lab computers and the computer lab computers, and there are instructions on how to install them on your own computer on the website. My instructions will assume you are running it on a lab computer, so the details may be different on your computer. G4beamline runs by reading a simple script. Like the MADX simulation, you can download the Main Ring example at the website, under the homework section. I'll assume that you rename it "lhc.g4bl" and then modify it for your purposes. Unlike MADX, G4Beamline uses full quad lengths, and places them at the full half-cell separation (i.e. no explicit "drifts"). Note that the units are a little weird: lengths are in mm, both for the dimensions and for the particle distributions. Angular distributions are in radians. Energy, etc, are in MeV, and magnetic gradients are in T/m, as usual.
    a. Modify the file to reflect the lengths, gradients, and particle distributions that you calculated for the LHC FODO cell in yesterday's homework.

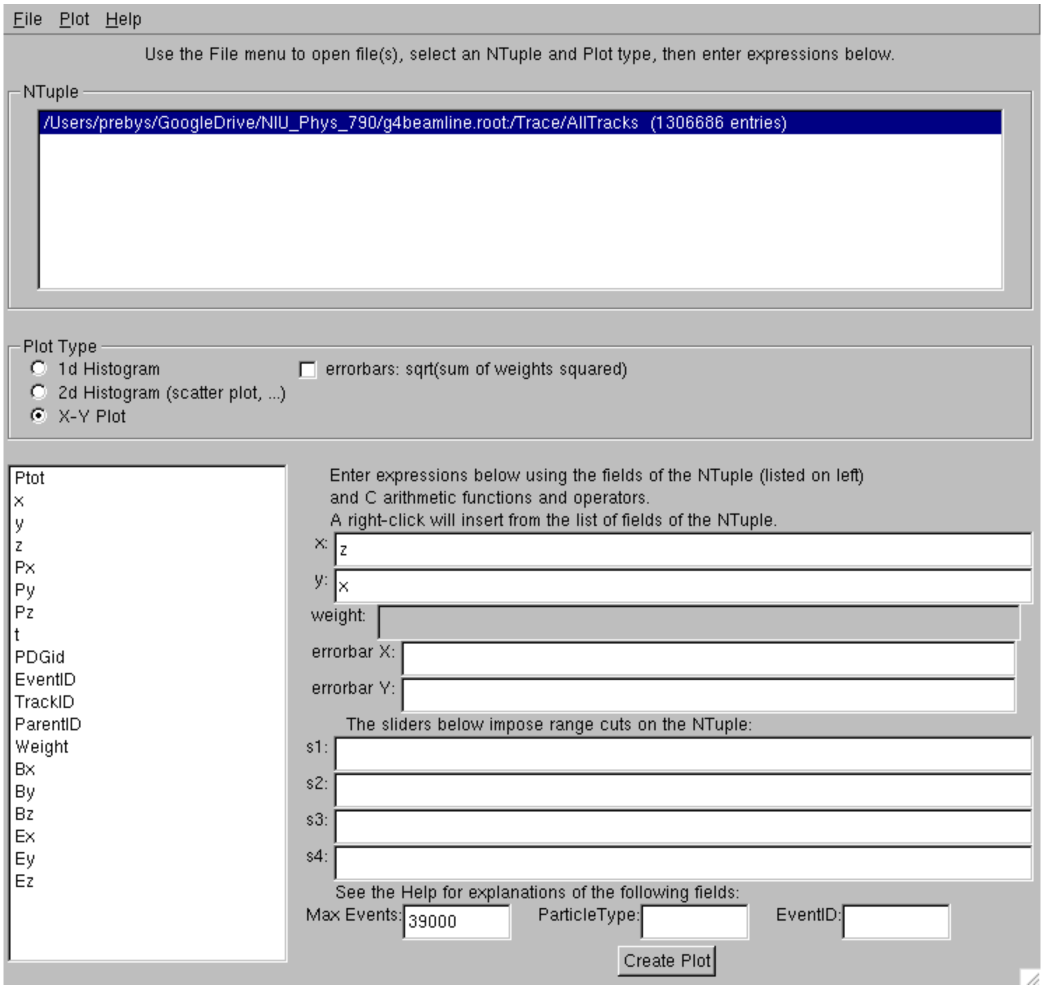
To run G4beamline, invoke the program from the desktop shortcut




The lab computers do not have the visualization option. If your computer has it, please uncheck it, as it will cause the simulation to run 1 event at a time. Browse to select your script file, and click "Run". It will generate 1000 events, which takes a couple of minutes.

The program will produce two files:

- g4beamline.root: This is a file containing tracking information for the first 100 tracks. There are roughly 13000 points per track for this case.
- profile.txt: This file has information on fitted distributions and regular intervals, including the lattice functions. The program doesn't understand beam optics, so it uses the techniques you established in problem 1 to calculate these values.



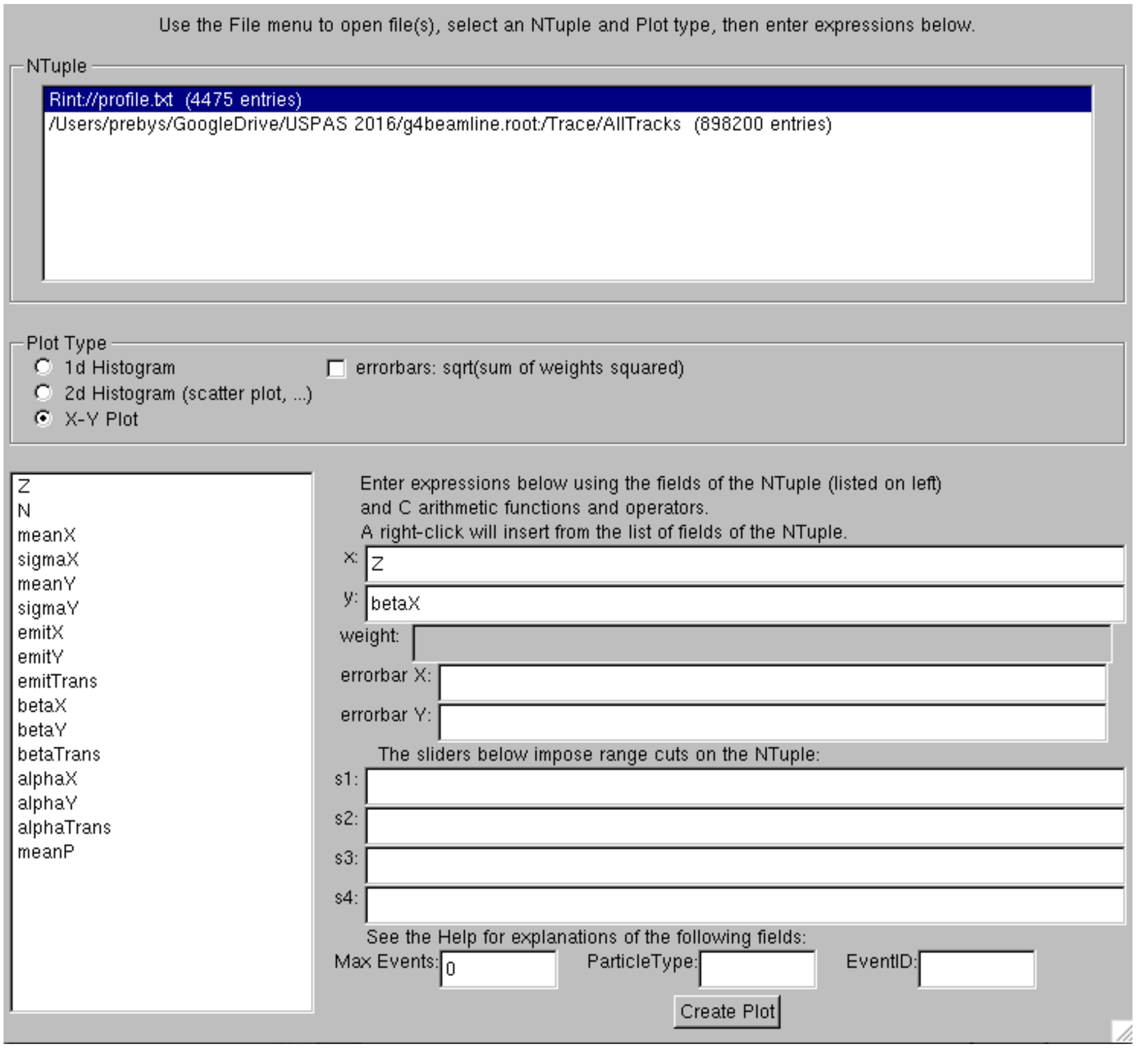

You can analyze these files with the "HistoRoot" program. Start it by clicking the icon. First, open the g4beamline.root file with the file browser[1]. You'll see an "Ntuple" in the list at the top. Select it, and the variables will appear at the left.

b. Select "X-Y Plot", and plot *x* vs. *z* (g4beamline uses *z* instead of *s*). Plot the first three events by setting "Max Events" to 39000 ("Events" in this case refer to the individual points on each track.)

c. Clear the "Max. Events" field to plot all points. If you did things right, you should see the periodic behavior of the beam envelope.

Now read in the profile.txt file with the file browser (you will have to explicitly select ".txt" in the file type to see it). A new NTuple "profile.txt" will appear in the list. Select it, and the new variables will appear at the left.

[1] For some reason, the lab computers generate an error at this point. Just keep clicking "Try Again", and you'll eventually get the file browser.




   d. Plot *betaX* vs. *Z* and verify that it matches your calculations (this *betaX* is in mm).
   e. Plot *alphaX* vs. *Z*.
   f. Plot *sigmaX* vs *Z* and compare the maximum and minimum to your calculations.
   g. Rerun the g4beamline program, but this time put "beamZ=-1550." on the Parameters line to start the beam at the beginning of the quad instead of the center. Repeat (b)-(f) to illustrate the effect that this mismatch has on the distributions.

3. All of our calculations assume that the nominal beam trajectory passes through the center of each quad, and it's very important to align the quads so that this happens. If a quad is out of alignment, the particles will see a net dipole term, which will distort the orbit. Using all the parameters that you have calculated[2] for the LHC at maximum energy (7000 GeV), answer the following questions:
   a. If one of the focusing quadrupoles is misaligned by 1 mm in $x$, what is the integrated dipole field [T-m] that particles will see? (just worry about the magnitude, not the sign.).
   b. How big of an angular deflection $\theta$ will this cause to the beam?

[2] By which I mean the values you calculated for yesterday's homework, *not* the values you measured in the simulation in part 2.

c. Based on our formula for closed orbit distortion (“Imperfections”, slide 4), what is the maximum deviation that this misalignment will cause in the ring? The LHC tune in the horizontal plane is 64.3. (hint: you don’t have to worry about specific phase advances. You can assume that at some points in the ring, the cosine term will be very close -1 or +1).