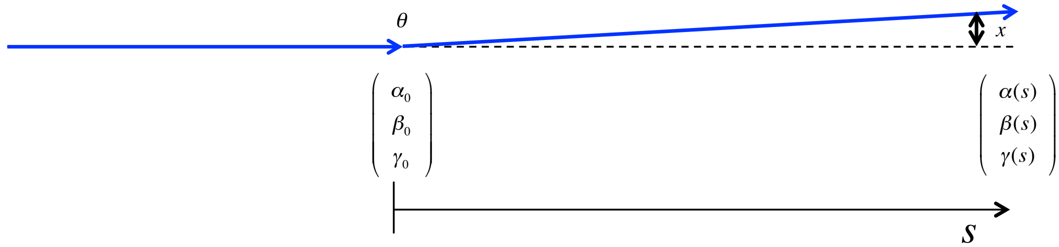
Accelerator Fundamentals Homework 3

1. Consider a particle traveling down the nominal trajectory through a drift (no dipoles or quadrupoles). A corrector dipole deflects it by an angle $\theta \ll 1$, as shown below



   a. Write the extremely simple expression for $x(s)$ in the world where you don't know anything about accelerator physics (i.e. not involving the lattice functions). As usual, you may assume $\tan\theta \approx \sin\theta \approx \theta$.

   b. Of course, we have shown you a much more complicated way to do the problem; namely (from Transverse Motion, slide 31)

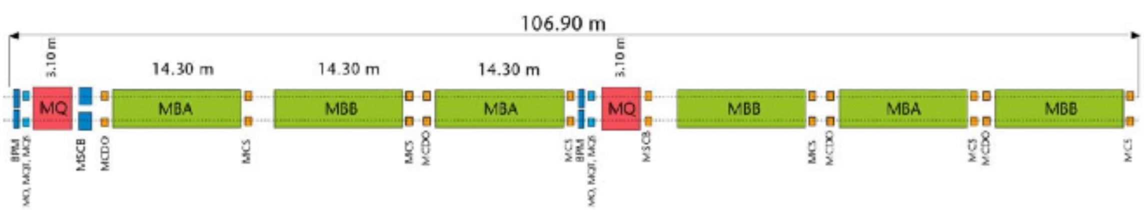
$$x(s) = \theta\sqrt{\beta_0\beta(s)}\sin\Delta\psi$$

   which looks very different. Are they the same? Let's see. Starting from an initial point $\beta_0 = 1\text{m};\ \alpha_0 = 0$, plot $\beta(s), \alpha(s), \gamma(s)$, and $\Delta\psi(s)$ vs. s for s from 0 to 10 m. You may use any plotting program you like (Excel, MathCad, etc).

   c. Use the result of (b) to calculate $x(s)$ for $\theta = 10\ \text{mrad}$ and show numerically that it's the same as the result from (a).

   d. Prove that whatever the initial β_0 is, both approaches give the same answer in general for the case of $\alpha_0 = 0$. (Hint: it will help you to remember, or prove to yourself, that $\sin(\tan^{-1} x) = x/\sqrt{1+x^2}$.)[1]

2. The LHC is has a design kinetic energy of 7000 GeV. The basic LHC FODO cell is shown below



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   It has a full length of 106.9 m ($L = 53.45\ \text{m}$), and a phase advance per cell of $\mu = 90^\circ$. Using the formulas we derived in class (Transverse Motion, slide 26), calculate the following:

   a. The focal length *f* of the quadrupoles required to give the specified phase advance [m].

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[1] You can of course prove the general case for $\alpha_0 \neq 0$, but it involves a pretty nasty integral. Try it if you enjoy that sort of thing.

b. If the quadrupoles are 3.1 m long, what gradient B' is required to achieve this focal length at full beam energy [T/m]
c. β_{max} at the center of the focusing quads [m].
d. $\beta_{\min}$ at the center of the defocusing quads [m].
e. The normalized RMS emittance of the beam is 2.75 μm. What is the physical transverse RMS of the beam (σ) in the middle of the focusing quads at the injection energy of 400 GeV [mm]?
f. What is the transverse RMS of the beam at the same location at the maximum energy [mm]?

3. Use the MADX simulation program to calculate the lattice parameters for the LHC FODO cell described in problem 2. You may run it on one of the lab computers, or download it to your own computer from http://cern.ch/madx .

   You can start with the MADX script I showed in class (see notes on page 43 of Transverse Motion). You may download the script that was shown in class and modify it for your needs. It is located on the website (http://home.fnal.gov/~prebys/misc/uspas_2016/) under the "Homework" section. You will need to change the length and strength of the quadrupoles, as well as the length of the drifts to match the LHC. Assuming you change the name of the file to "lhc.madx", the program is run by typing "madx < lhc.madx"[2]. It will generate two files: a PostScript graphics file with the lattice functions plotted, and a text file, with the lattice parameters at various points. Note that the lattice values are given at the *end* of the element listed, so for example, you would find the lattice values for the middle of a quad listed on the line for the first half.

   Compare the values for β_{max} and β_{min} that are calculated by MADX to the ones you calculated in problem 2.

[2] You may have to explicitly specify the path. For example, on the lab computers, you will have to type: "\Program Files (x86)\mad\madx\madx.exe" < lhc.madx (with the quotation marks).