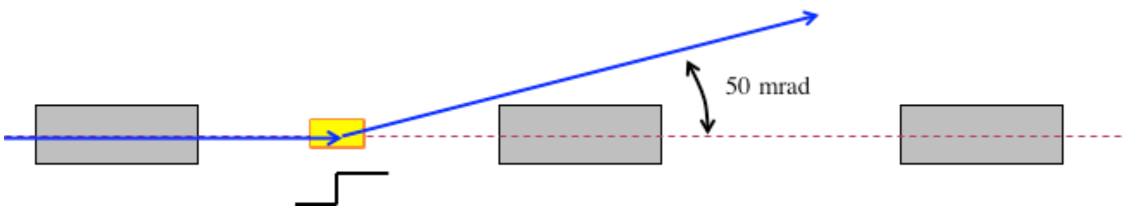
Accelerator Physics Homework 2

1. For a proton beam with kinetic energy of 10 GeV, calculate:
    a. The total energy [GeV]
    b. The momentum [GeV/c]
    c. The velocity [fraction of *c*]
    d. The rigidity $(B\rho)$ [T-m]
2. This beam is circulating in a synchrotron, and we wish to extract it by inserting a small pulsed magnet in a straight section, as shown below.



In order to clear the next magnet, we need to bend the beam by at least 50 mr.
    a. If we use a 1m long dipole magnet, what field [T] will be required?
    b. In class, we calculated the field of a dipole magnet as

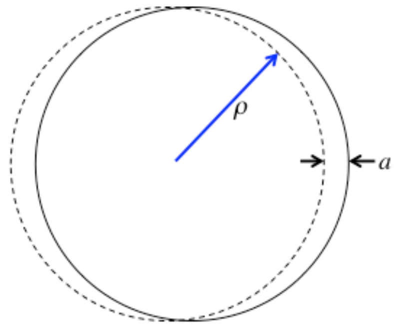
$$B = \frac{\mu_0 IN}{g}$$

show that if the length and width of the pole face are *l* and *w*, the inductance is

$$L = \frac{\mu_0 N^2 wl}{g}$$

(reminder: inductance is defined as total magnetic flux divided by current)
    c. In order for the beam to fit, our 1 m long extraction magnet has to have *g*=*w*=5cm. To keep inductance low, we use a single turn (that is, *N*=1)
        i. What is the inductance of the magnet [H]?
        ii. What current will be required [A]?
        iii. The beam is circulating, so we need a very fast rise time. If we assume the current rises linearly to the required value in 50 ns, what will be the inductive voltage $\left(V = L\frac{dI}{dt}\right)$ on the magnet [V]? (note: if you did the problem correctly, you'll get an *extremely* large value here).
3. Consider a particle with a momentum of p in a uniform magnetic field B. As we have shown, it will move in a circle with a radius of curvature $\rho = p/qB$. If we offset the particle from the

reference orbit, as shown below, it will of course move in a circular path, just with a different center



On the other hand, if we think of this in terms of our accelerator formalism, this can be considered oscillatory motion about the reference orbit (shown as a dotted line). Show that this is the same answer you get if you start with the equation of motion

$$x'' + \left[\frac{1}{\rho^2} + \frac{1}{(B\rho)}\frac{\partial B_y(s)}{\partial x}\right]x = 0$$

that we derived on p. 17 of the "Transverse Motion" lecture, and solve for the motion in x (note: you don't have to show that the functional form for x is the same; only that you get the correct period and amplitude.). You may neglect the fact that motion is unstable in y .