Accelerator Fundamental Homework 1

Here are some general warm-up problems to practice some of the skills which will be required.

1. A matrix operation is defined in the XY plane as

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \equiv \mathbf{M} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

   In a rotated coordinate system where

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \equiv R \begin{pmatrix} x \\ y \end{pmatrix}$$

   show that

$$\begin{pmatrix} x_1' \\ y_1' \end{pmatrix} = \mathbf{M}' \begin{pmatrix} x_0' \\ y_0' \end{pmatrix}; \text{ where } \mathbf{M}' = \mathbf{RMR}^{-1}$$

   write expressions for the elements a', b', c' and d' of $\mathbf{M}'$ in terms a, b, c, d and θ.

2. Evaluate the following integral

$$\int r^3 z^2 dV$$

   over a volume defined by a cylinder of length L and radius R, oriented along the z axis and centered at the origin. Hint: first integrate over a circle, and then integrate along the *z* axis.

3. Given the usual definitions of β and γ, prove the following relationships, which we will use a lot:

$$d\gamma = \beta\gamma^3 d\beta$$
$$\frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dp}{p}$$
$$\frac{dp}{p} = \frac{1}{\beta^2}\frac{dE}{E}$$

4. The Fermilab Linac produces a proton beam with kinetic energy of 750 MeV. Calculate the velocity (in m/s), Energy (in MeV) and momentum (in MeV/c) for this beam.