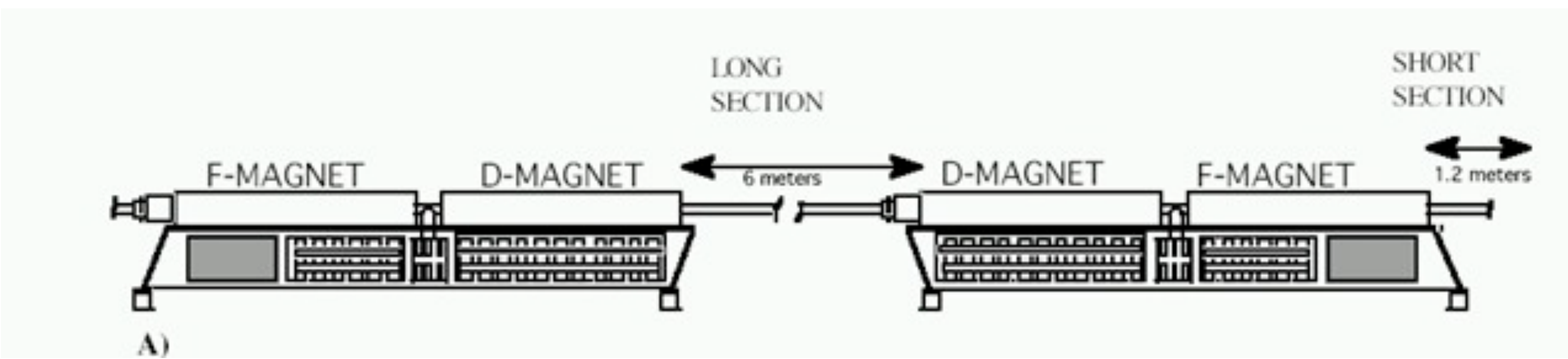
Insertions
(Complete Version)

The Problem

- So far, we have talked about a synchrotron made out of identical FODO cells, with the space between the quads taken up by bend dipoles.
- The problem is that this is not particularly useful, because there's no place to put beam in or take it out, and no way to collide beams.
- One solutions is to design a "straight" into every cell. Example: the Fermilab Booster

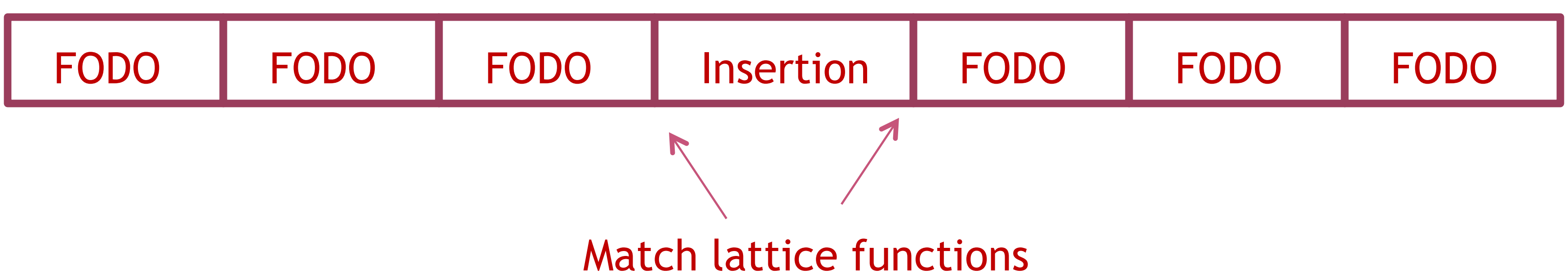


x24

- However, this is very wasteful of real estate. It would not be practical for the LHC.

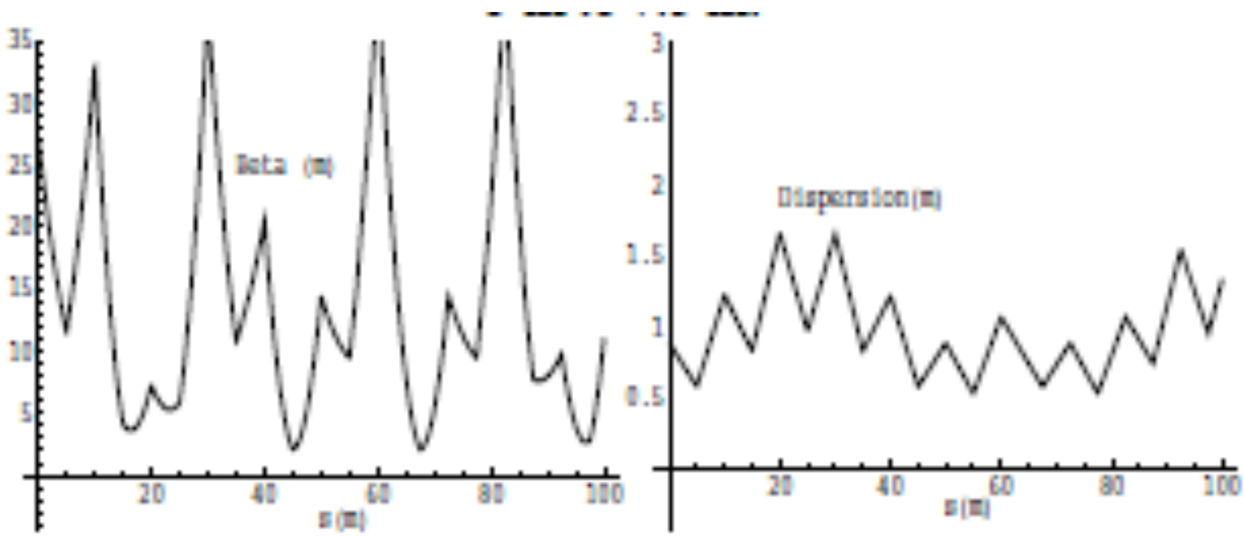
Insertions

- Since putting a empty straight section in every period is not practical, we need to explicitly accommodate the following in our design:
  - Locations for injection of extraction.
  - “Straight” sections for RF, instrumentation, etc
  - Low beta points for collisions
- Since we generally think of these as taking the place of things in our lattice, we call them “insertions”



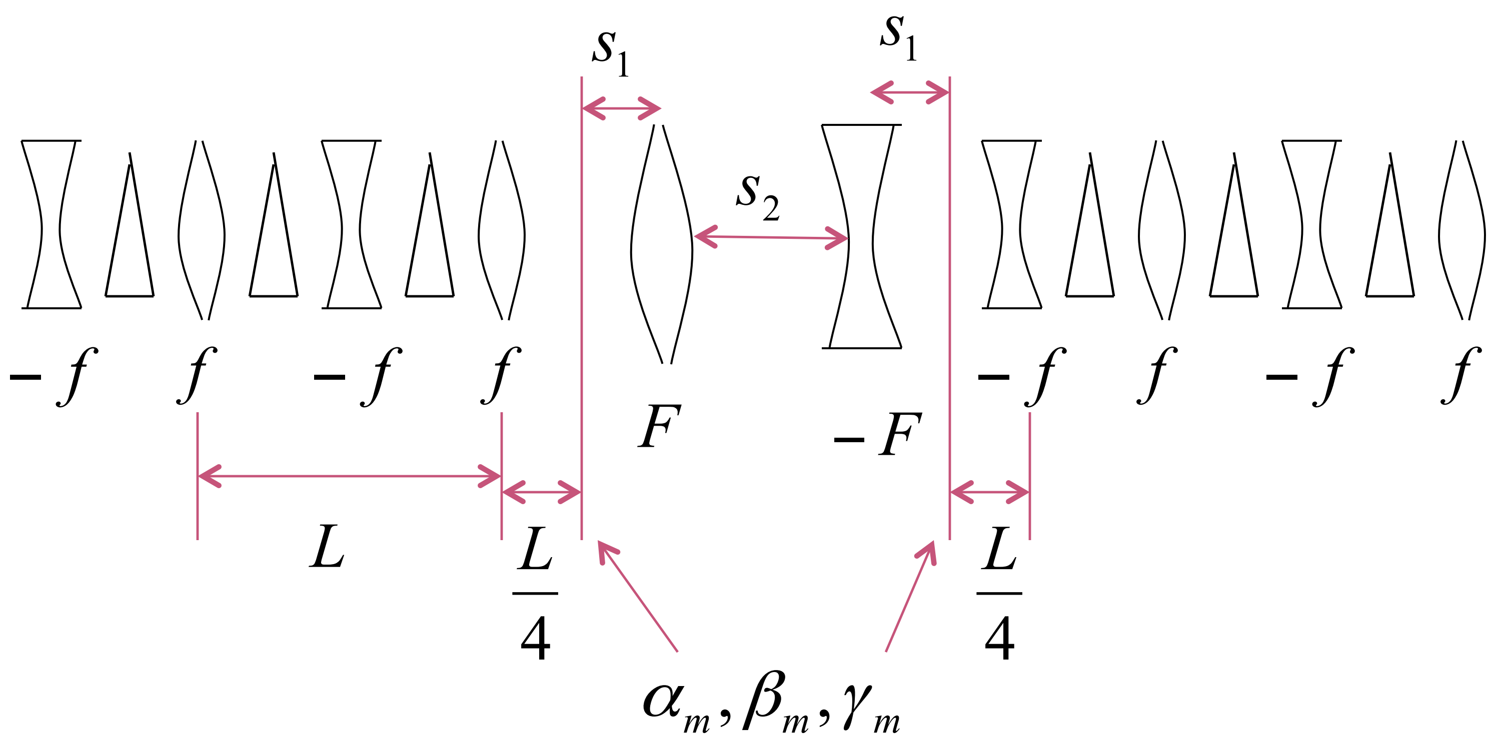
Mismatch and Beta Beating

- Simply modifying a section of the lattice without matching will result in a distortion of the lattice functions around the ring (sometimes called “beta” beating)
- Here’s an example of increasing the drift space in one FODO cell from 5 to 7.5 m



Collins Insertion

- A Collins Insertion is a way of using two quads to put a straight section into a FODO lattice



- Where s_2 is the usable straight region

- Require that the lattice functions at both ends of the insertion match the regular lattice functions at those point

$$M=\begin{pmatrix}1 & s_1\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{1}{F} & 1\end{pmatrix}\begin{pmatrix}1 & s_2\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -\frac{1}{F} & 1\end{pmatrix}\begin{pmatrix}1 & s_1\\ 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}\cos\mu_I+\alpha_m\sin\mu_I & \beta_m\sin\mu_I\\ -\gamma_m\sin\mu_I & \cos\mu_I-\alpha_m\sin\mu_I\end{pmatrix}$$

Where μ_I is a free parameter

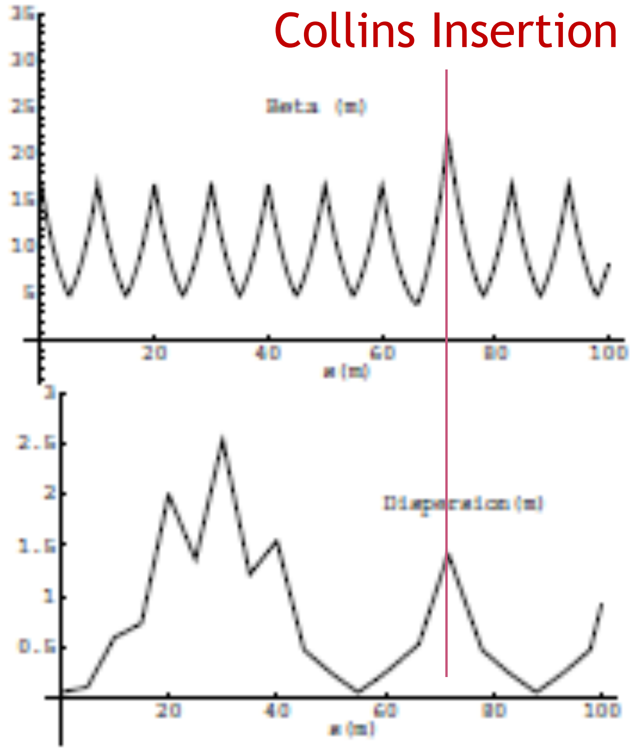
- After a bit of algebra

$$s_1=\frac{\tan\frac{\mu_I}{2}}{\gamma};s_2=\frac{\alpha^2\sin\mu_I}{\gamma};F=-\frac{\alpha}{\gamma}$$

- Maximize s_2 with $\mu_I=\pi/2$, α max (which is why we locate it *L*/2 from quad)
- Works in both planes if $\alpha_x=-\alpha_y$ (true for simple FODO)

Dispersion Suppression

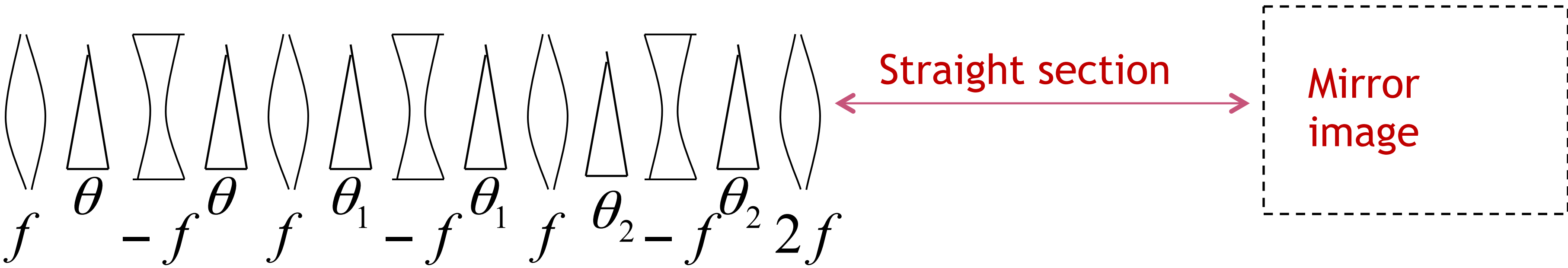
- The problem with the Collins insertion is that it does *not* match dispersion, so just sticking it in the lattice will lead to distortions in the dispersion



- This is typically dealt with by suppressing the dispersion entirely in the region of the insertion.

Dispersion Suppression (cont'd)

- On common technique is called the "missing magnet" scheme, in which the FODO cells on either side of the straight section are operated with two different bending dipoles and a half-strength quad



- Recall that the dispersion matrix for a FODO half cell is (lecture 4)

$$\mathbf{M}=\begin{pmatrix}1 & 1 & 0\\ -\frac{1}{2f} & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & L & \frac{L\theta}{2}\\ 0 & 1 & \theta\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 1 & 0\\ \frac{1}{f} & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & L & \frac{L\theta}{2}\\ 0 & 1 & \theta\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 1 & 0\\ -\frac{1}{2f} & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}1-\frac{L^2}{2f^2} & 2L\left(1+\frac{L}{2f}\right) & 2L\theta\left(1+\frac{L}{4f}\right)\\ -\frac{L}{2f^2}+\frac{L^2}{4f^3} & 1-\frac{L^2}{2f^2} & 2\theta\left(1-\frac{L}{4f}-\frac{L^2}{8f^2}\right)\\ 0 & 0 & 1\end{pmatrix}$$

- So we solve for

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \mathbf{M}(\theta = \theta_2)\mathbf{M}(\theta = \theta_1)\begin{pmatrix} D_m \\ D'_m \\ 1 \end{pmatrix}$$

- Where D_m and D'_m are the dispersion functions at the end of a normal cell (for a simple lattice, $D'_m=0$)
- We get the surprisingly simple result

$$\theta_1 = \theta\left(1 - \frac{1}{4\sin^2\frac{\mu}{2}}\right); \theta_2 = \theta\frac{1}{4\sin^2\frac{\mu}{2}}$$

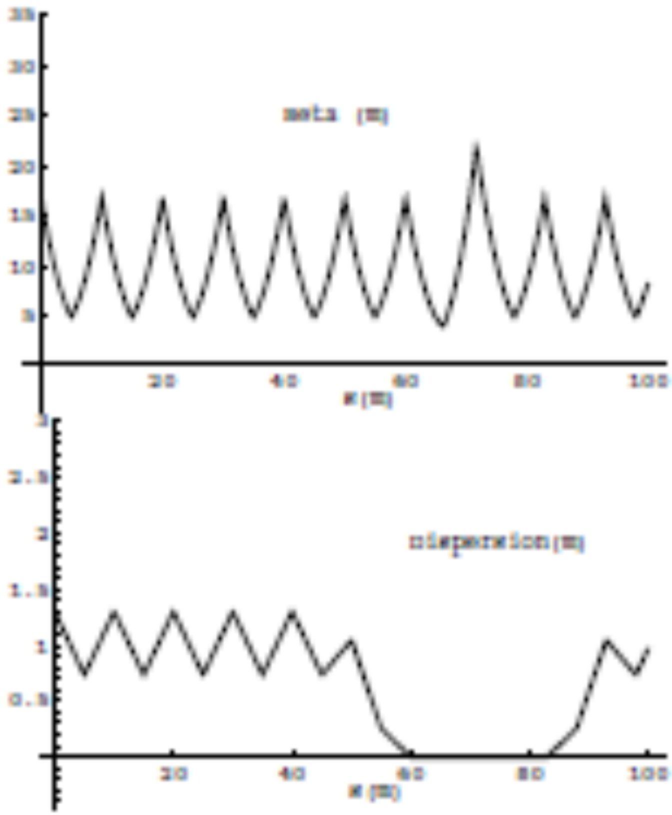
- Note that if $\theta=60^\circ$

$$\theta_1 = 0 \qquad \theta_2 = \theta$$

- So the cell next to the insertion is normal, and the next one has no magnets, hence the name "missing magnet".
- Since dispersion can only be generated by bend magnets, if I suppress it before a straight section, it will remain zero in the straight section

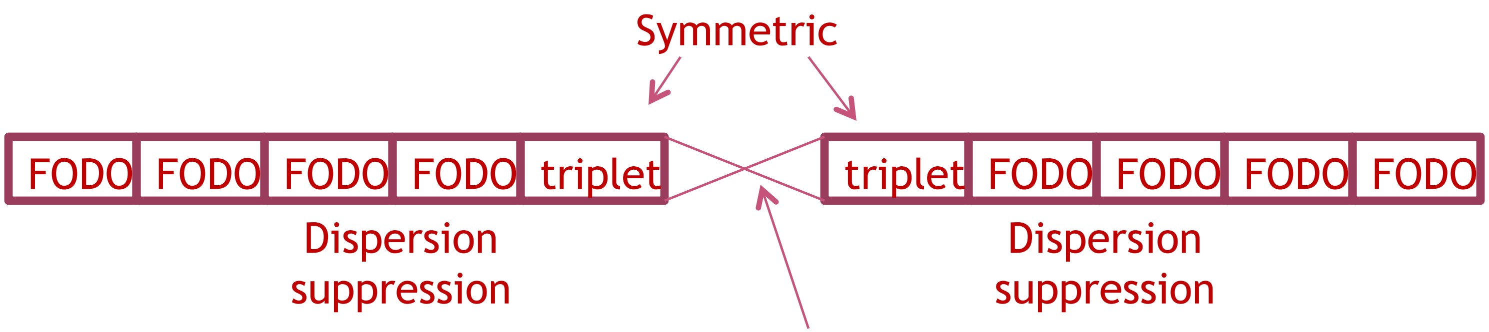
Combining Insertions

- Because the Collins Insertion has no bend magnets, it cannot generate dispersion if there is none there to begin with, so if we put a Collins Insertion inside of a dispersion suppressor, we match both dispersion and the lattice functions.



Low β Insertions

- In a collider, we will want to focus the beam in both planes as small as possible.
- This can be done with a symmetric pair of focusing triplets, matched to the lattice functions (dispersion suppression is assumed)



$$\beta = \beta^*; \alpha = 0$$

- Recall that in a drift, β evolves as

$$\beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2 = \beta^* + \frac{s^2}{\beta^*}$$

Where *s* is measured from the location of the waist

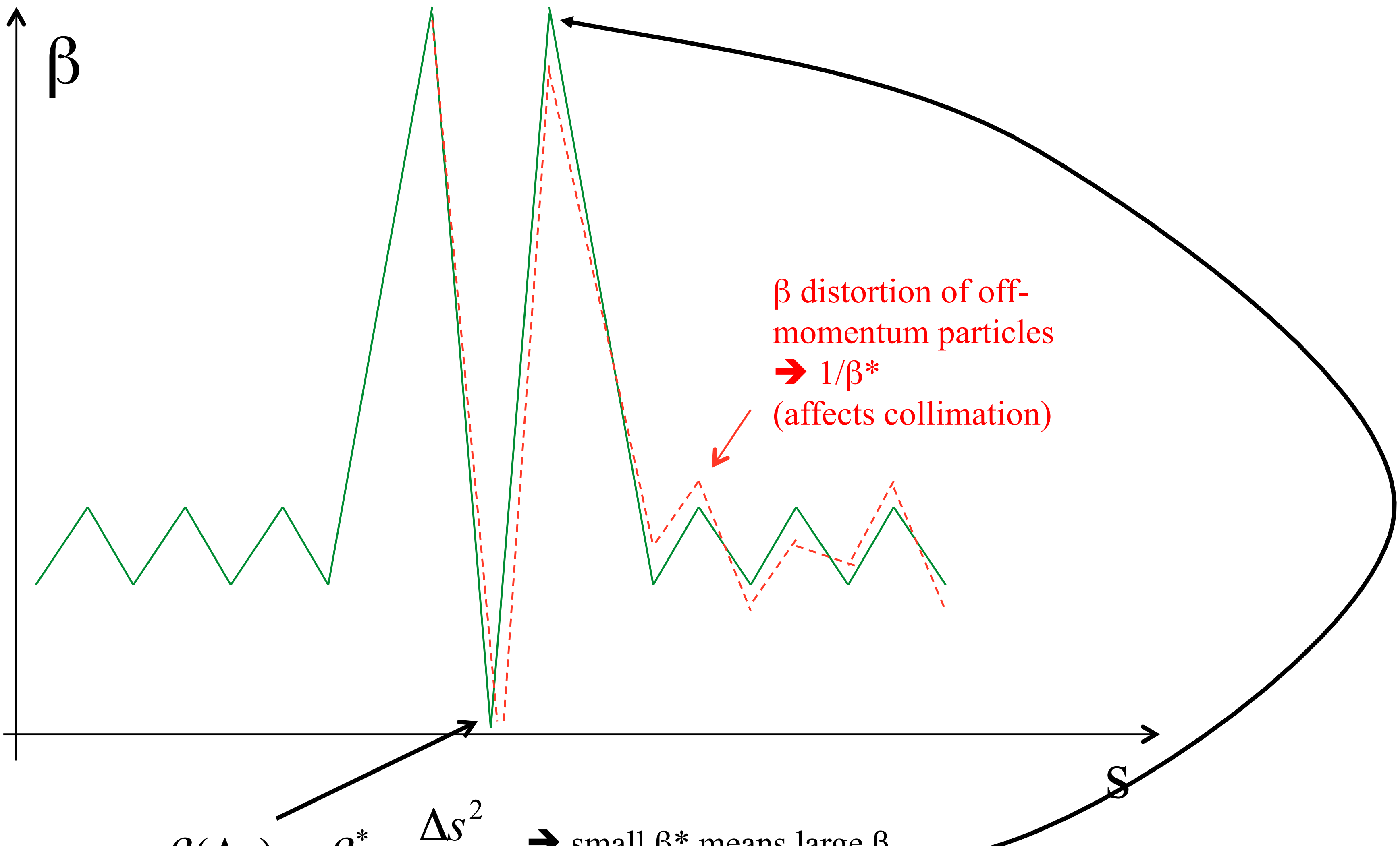
Phase Advance of a Low Beta Insertion

- We can calculate the phase advance of the insertion as

$$\Delta\psi = \int_{-L/2}^{L/2} \frac{ds}{\beta} = \frac{1}{\beta^*} \int_{-L/2}^{L/2} \frac{ds}{\left(1 + \left(\frac{s}{\beta^*}\right)^2\right)} = 2\tan^{-1}\left(\frac{L}{\beta^*}\right)$$

- For L>>β*, this is about π, which guarantees that all the lattice parameters will match except dispersion (and we've suppressed that).
- This means that each low beta insertion will increase the tune by about 1/2

Limits to β*



$$\beta(\Delta s) = \beta^* + \frac{\Delta s^2}{\beta^*}$$

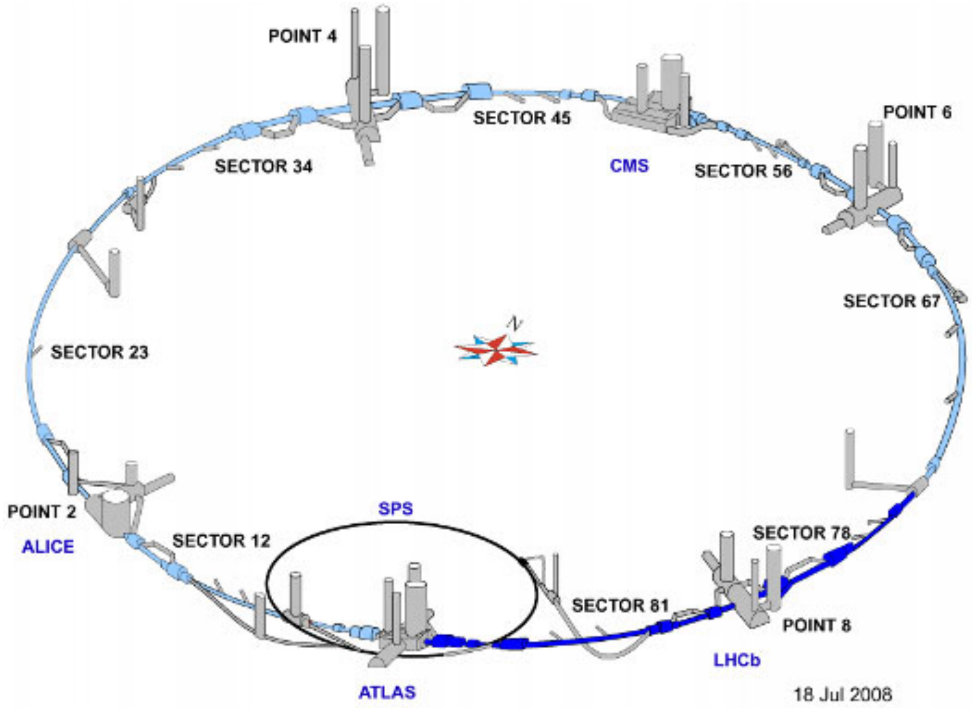
➔ small β* means large β (aperture) at focusing triplet

Putting the Pieces Together

- So now we see that in general, a synchrotron will contain
  - A series of identical FODO cells in most of the ring.
  - Straight sections, with modified cells on either end.
  - Dispersion suppression before and after these straight sections
- If it's a collider, it will also contain
  - One or more low beta insertions with dispersion suppression on either side.
  - The beta function will be very large on either side of the low beta point

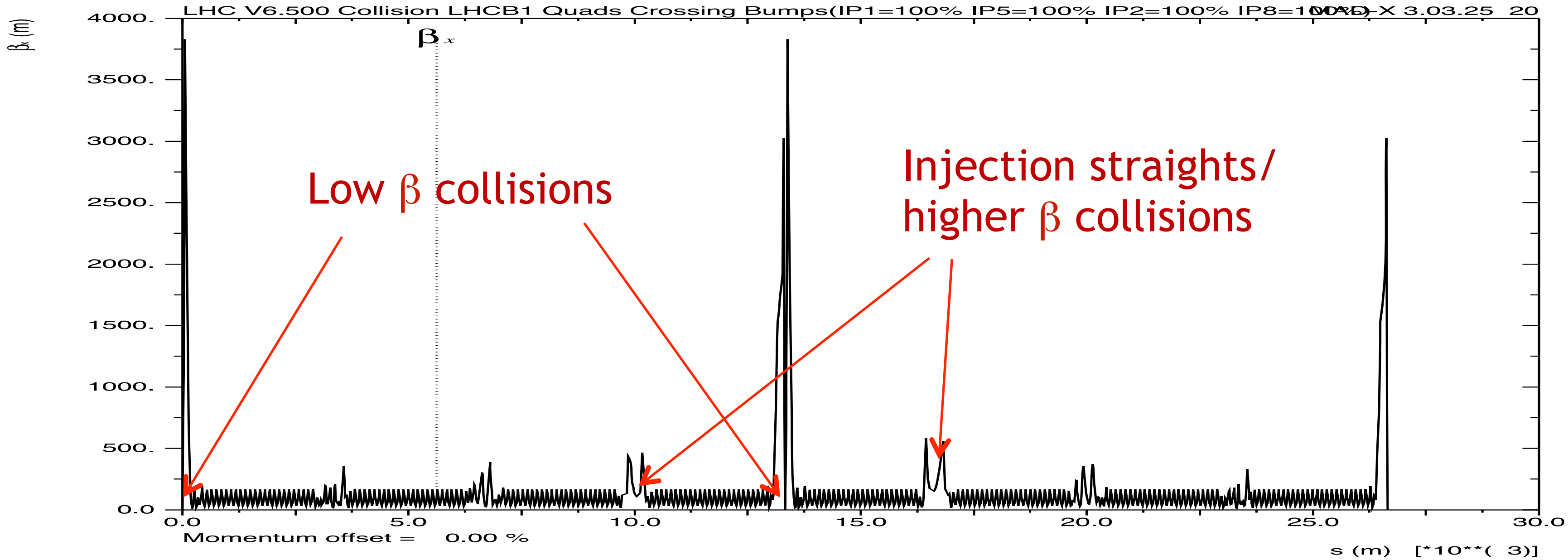
Example: LHC

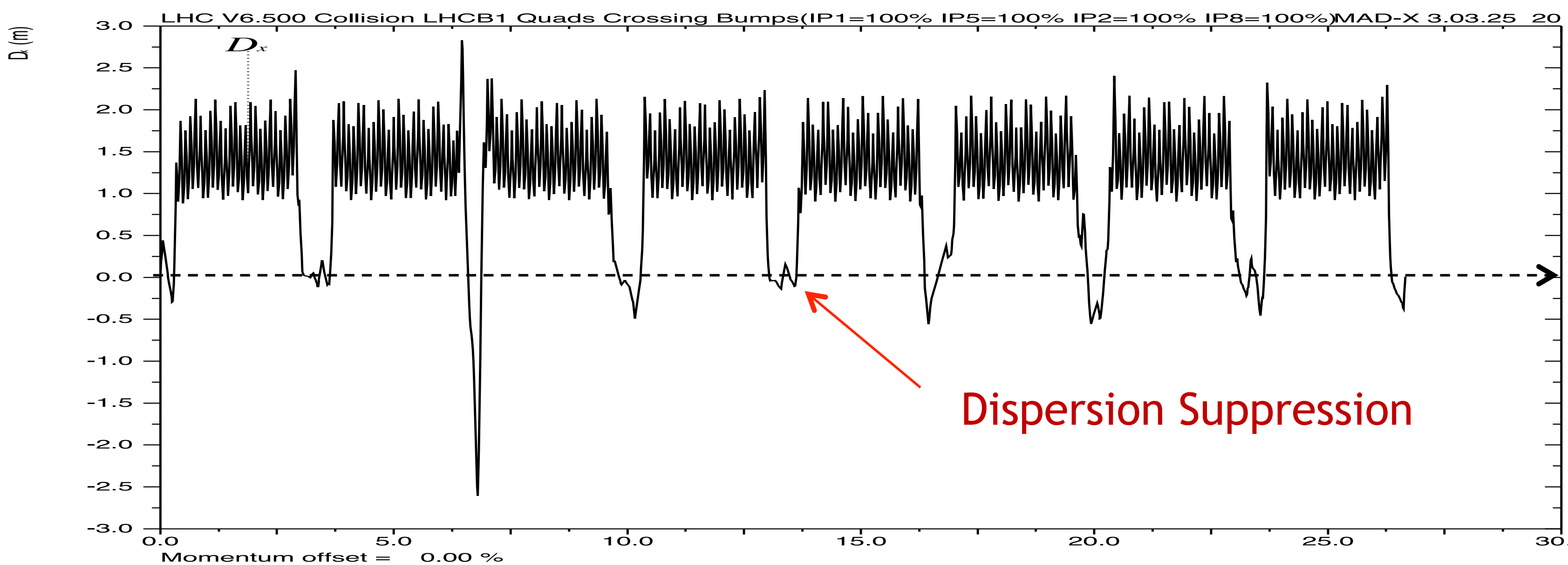
- Recall the LHC layout



  - Superperiodicity of 8
  - Need insertions for two low beta collision regions (ATLAS, CMS)
  - Two higher beta collision regions (ALICE, LHCb), which double as injections points.
  - Other straights for RF cavities, beam extraction, etc.

LHC Optics (out of date)



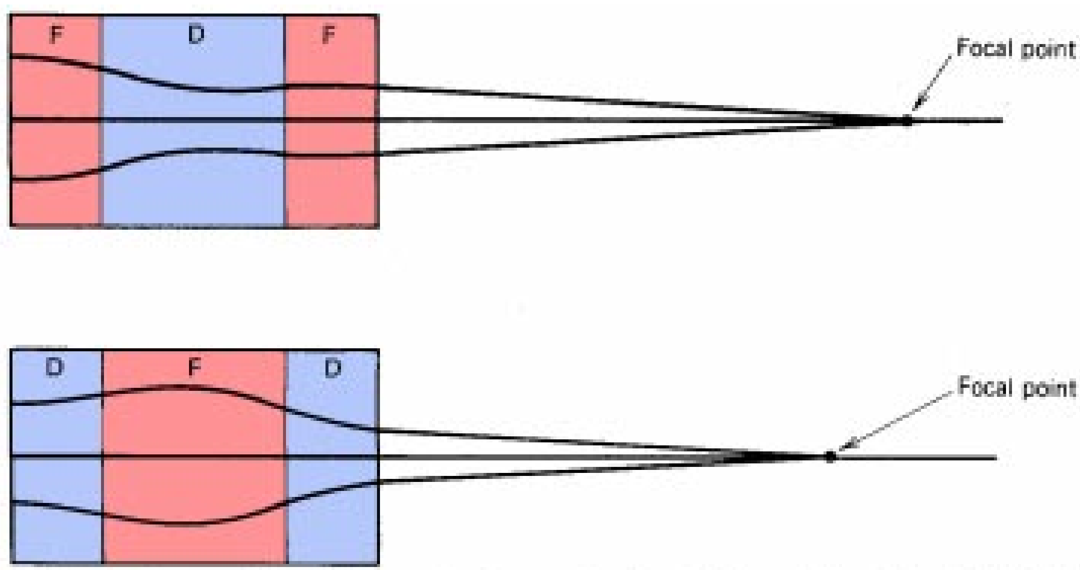


Beam Line Issues (not in original printout)

- Beam lines are typically built in discrete sections:
  - Matching (to a source, injection point, or extraction point)
  - Transport:
    - The FODO cells we've been talking about
  - Bends
    - Designed as "achromats" to suppress dispersion!
  - Focus (or "waist")
    - Uses quad triplet to minimize beta in both planes
  - Collimation sections
    - 90° apart in phase space to clean up 2D phase space

Final Focus Triplet

- As we saw, our normal FODO cell has maxima in one plane where the minima are in the other.
- For targets or collisions, we want small beta functions in both planes.
- This optical problem can be solved with a triplet
  - Middle quad ~twice the strength of outer quads (MAD problem for next week)



Dispersion Suppression

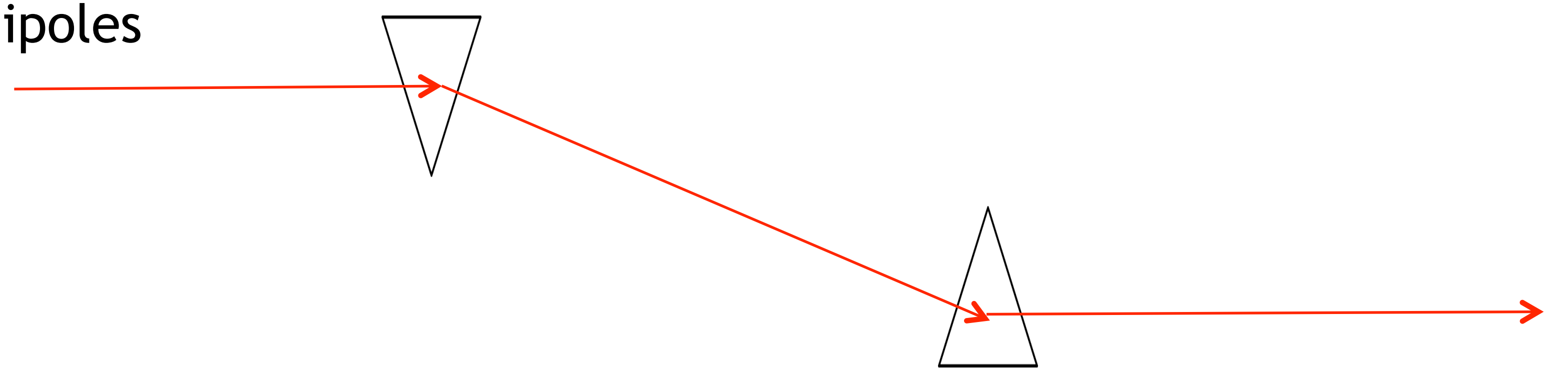
- Any bend section will introduce dispersion. After the bend, it will propagate as

$$\begin{pmatrix} D_x(s) \\ D'_x(s) \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D_x(0) \\ D'_x(0) \\ 1 \end{pmatrix}$$

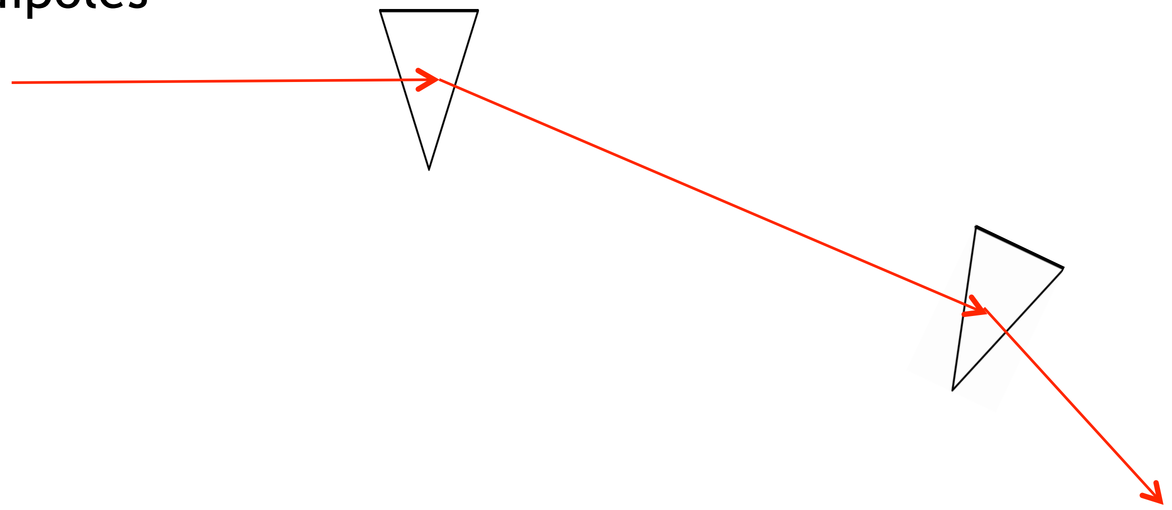
- It will never go away unless we explicitly suppress it in the design

Achromats

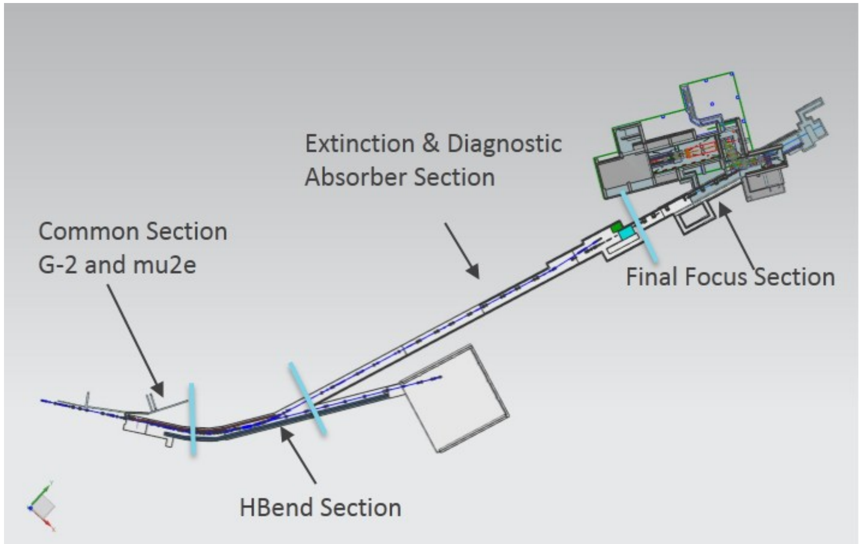
- There are generally two types of *basic* "achromats"
- "Dogleg", with 360° of phase advance between to opposite sign dipoles

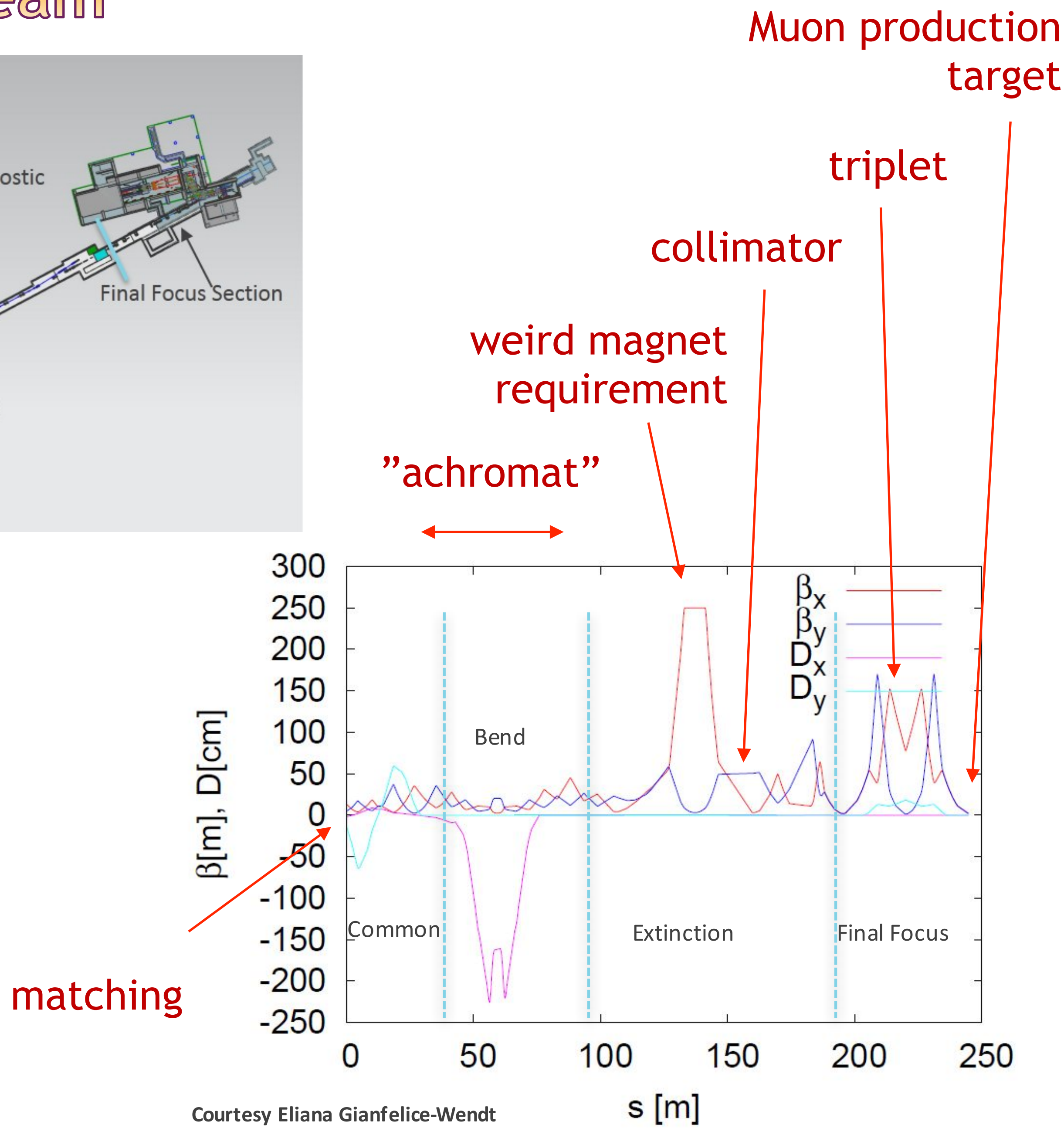


- "Double bend", with 180° of phase advance between two same sign dipoles



Mu2e Proton Beam





Courtesy Eliana Gianfelice-Wendt

G4beamline version

- Converted from MAD file with (homebrew) Python Script