



# Fundamentals of Accelerator Physics and Technology with Simulations and Measurements Lab

U.S. Particle Accelerator School  
13 - 24 June 2016

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## Computer Session – Emittance Issues: Synchrotron Radiation & Matching

In this session, we will look at emittance issues, particularly unique ones pertaining to synchrotron radiation as well as common issues of emittance matching between systems. Two Mathcad<sup>1</sup> worksheets will be used to aid in our investigation. A few hints on using *Mathcad* are found on the last page. Questions to be answered and turned in for grade are typeset in **Bold Type**

### 1 Synchrotron Radiation

Open the Mathcad worksheet ***SynchLight.mcd***. This worksheet lays out a 10 GeV electron synchrotron of circumference 600 meters. Scroll through the upper part of the worksheet to appreciate the input parameters to set up our accelerator.

Two plots are generated at the bottom. The first shows the rms transverse beam sizes as a function of time, and the other plots the rms energy spread as a function of time. Note that damping occurs over a short time (see the x-axis units).

In class we learned/will learn that the damping times are all related, as summarized by Robinson's Theorem:

$$1/\tau_x + 1/\tau_y + 1/\tau_s = 2/\tau_0.$$

For reference, refer to *Syphers/Edwards chapter 8: Synchrotron Radiation*. Look at the plot of  $\sigma_x$  and  $\sigma_y$ . Is the 'macro' behavior in each plane as you would expect over time?

- Q1) Determine how long it takes the beam sizes and energy spread to each reach equilibrium. How does this compare to the damping times?**
- Q2) Compare the damping times of the three plotted parameters. Do they relate to each other as expected?**
- Q3) The emittance is related to  $\sigma^2$ . Adjust the first plot to display  $\sigma$  (the default) and  $\sigma^2$  for x. How does the emittance damping time compare to the damping time for  $\sigma_x$  (the beam size).**

<sup>1</sup>Mathsoft, Inc.  
Emittance Issues

- Q4) The worksheet calculates the equilibrium emittance. At a  $\beta_{\max}$  location, what would be the horizontal beam size (in millimeters) just due to emittance?**
- Q5) What would be the total horizontal beam size taking Dispersion into account as well?**

## 2 Emittance Issues

Large accelerator complexes often are composed of a chain of machines with ever-increasing final energies. It can be challenging to efficiently transfer beams from one accelerator to another while preserving the emittance. Mismatches in the  $\beta$ -function, steering errors, and thin foils used to isolate vacuum between one ring and another, for example, can all be sources of emittance dilution (growth). In this exercise we look at the effect each of these can have on our particle beam.

Open the Matchcad worksheet, *Dilution.mcd*. By inputting an injection offset,  $\Delta x$ , or a beta function mismatch,  $r_\beta$  and looking at the behavior of  $N$  particles, 1000 for this case, over an certain number of turns,  $N_t$ , we can evaluate the emittance growth.

The output of the worksheet is two plots. The first one displays the phase space distribution of the  $N$  particles after  $N_t$  turns. The y-axis plots  $px = \beta x' + \alpha x$ , and the x-axis simply displays  $x$ . The other graph displays a histogram of the particle distribution in  $x$ . Also shown are the mean value and standard deviation for  $x$ . Calculated values of the emittance increase, predicted steering mismatch, and amplitude function mismatch are found at the bottom.

Let's begin with baseline (ideal) conditions: for  $\Delta x = 0$  (no position offset),  $r_\beta = 1.0$  (no beta function mismatch). For  $N_t = 0$  record: mean  $x_f$ ,  $\text{stdex}(x_f)$ , Emittance increase, Steering Mismatch, and Amplitude function mismatch below.

- Q6) Set  $N_t = 100$ . How do the 'mean( $x_f$ )', 'stdev( $x_f$ )', 'Emittance Increase', and the predicted values of 'Steering Mismatch', and 'Amplitude function Mismatch' compare with the  $N_t = 0$  case?**

mean( $x_f$ ) = \_\_\_\_\_ stdev( $x_f$ ) = \_\_\_\_\_

Emittance Increase = \_\_\_\_\_

Steering Mismatch = \_\_\_\_\_

Amplitude function Mismatch = \_\_\_\_\_

Return  $N_t$  to 0. We will now analyze the effect of a steering (displacement) error. Set  $\Delta x = 2$  (sigma).

**Q7) What values of these parameters are generated for  $N_t = 0$ ?  $N_t = 100$ ? How many turns does it take for the emittance to reach equilibrium in this situation?** *hint – look at the trend of how the emittance increases between 0 and 100 turns*

mean( $x_f$ ) = \_\_\_\_\_ stdev( $x_f$ ) = \_\_\_\_\_

Emittance Increase = \_\_\_\_\_

Steering Mismatch = \_\_\_\_\_

Amplitude function Mismatch = \_\_\_\_\_

Turns = \_\_\_\_\_

Let's repeat this for a beta function mismatch. Here, the parameter " $\delta\beta$ " is really  $\Delta\beta/\beta$ . So, take  $r_\beta = 2.0$ . This makes  $\delta\beta = 1$  which is a 100%(!) mismatch of the incoming  $\beta$  function.

**Q8) What values of these parameters are generated for  $N_t = 0$ ?  $N_t = 100$ ? How many turns does it take for the emittance to reach equilibrium for  $r_\beta = 2.0$ ?** *hint – look at the trend of how the emittance increases between 0 and 100 turns*

mean( $x_f$ ) = \_\_\_\_\_ stdev( $x_f$ ) = \_\_\_\_\_

Emittance Increase = \_\_\_\_\_

Steering Mismatch = \_\_\_\_\_

Amplitude function Mismatch = \_\_\_\_\_

Turns = \_\_\_\_\_

**Q9) Which effect has the greater impact on emittance growth?**

It is possible to model the beam passing through a foil by modifying the  $x$  and  $p$  input parameters. Replace  $\sqrt{r_p}$  with 1 and  $1/\sqrt{r_p}$  with, say,  $\sqrt{1+1^2}$ , ( $r_p$  must be  $>0$ ) respectively.

Look at the phase space distribution for  $N_t = 0$ . The distribution appears to be 'standing up'. When a beam passes through a foil, scattering occurs which increases the angular distribution while the beam width stays fixed. Over successive turns the distribution will tumble, finally reaching equilibrium.

**Q10) What values are generated for  $N_t = 0$ ?  $N_t = 100$ ? How many turns does it take for the emittance, in this case, to reach equilibrium?**

*Same hint...*

mean( $x_f$ ) = \_\_\_\_\_      stdev( $x_f$ ) = \_\_\_\_\_

Emittance Increase = \_\_\_\_\_

Steering Mismatch = \_\_\_\_\_

Amplitude function Mismatch = \_\_\_\_\_

Turns = \_\_\_\_\_

### 3 Some General *Mathcad* Hints

- To change a parameter, simply click on it until a cursor appears to its right, and then delete or alter it to how you want it. When you hit RETURN, Mathcad will update the entire worksheet.
- If you wish to update the worksheet manually, deselect Automatic Calculation in the MATH menu.
- If you wish to recalculate the worksheet without moving the cursor, just hit “Control-=”.
- If you wish to move an equation, expression, or text area around, first click on the item. Put the cursor near the edge of the item’s surrounding box; a hand should appear. Now you can drag the item to its new location.
- Mathcad has a Help feature which can be obtained by selecting HELP TOPICS under the “?” icon in the upper right hand corner of the screen. There is also a “?” icon in one of the tool palettes.
- An expression like “y:=x” is a definition of y in terms of x. An expression like “y=0.56” is the result of a calculation of y.
- To terminate a calculation, press “Control-.” (Control-period) to abort the process. One can also hit the “Escape” key.