USPAS Accelerator Physics Final Exam

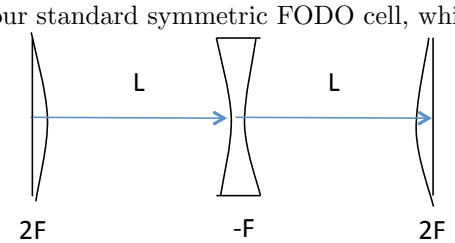
January 29, 2015

This copy of the final includes solutions, shown in red. Full derivations are given, along with example sources for the necessary equations. The requested answers are $\boxed{\text{boxed}}$. The number of possible points for each section of each problem is shown, with a total of 70 points being possible for the exam. Generally, if an error in one calculation results in errors in subsequent calculations, full credit will be given for the later calculations, provided their answers are consistent with the earlier (ie, incorrect) value.

General Guidelines

- This is an "open book" exam. You may use the texts, lectures, homeworks, or any of the online resources, including previous finals. You are expected to work independently and to not seek out other sources for the solutions.
- There are a total of three problems, which do not have equal weight.
- You may use anything that appeared in the lectures, textbook or assigned homework, without re-deriving it.
- Full or partial credit will only be given if your reasoning can be followed, so show your work. Please give answers in the requested [units] when specified.
- The exam is due at 9AM tomorrow. Late exams will have their score reduced by 10%, with an additional 10% deducted for each additional hour.
- All problems are straightforward applications of what you have learned. There are no trick questions or complex calculations. If you find yourself working hard, it's a good sign you're not doing the problem correctly.
- If you think there's a problem with the test, contact Eric at 630-336-1893 or prebys@fnal.gov. Any necessary corrrections or clarifications will be sent to the email list and posted on the web page, so check both frequently!

All three problems will be based on our standard symmetric FODO cell, which we have seen many times:

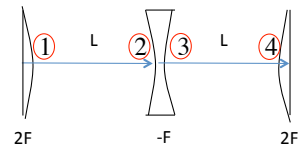


where each cell contains focusing and defocusing quadrupoles of focal lengths (in the horizonal plane) of F and $-F$, respectively, spaced L apart . Positions s within the cell are measured from the center of the first focusing quadrupole. You may use the usual thin lens approximation for the quadrupoles. In all cases, "focusing quadrupole" and "defocusing quadrupole" will refer to the quadrupoles which focus or defocus, respectively, in the bend (x) plane.

Problem 1 (18 points total)

a. Write expressions for all Twiss parameters in the bend plane (α_x,β_x, and γ_x) immediately before and immediately after each focusing and defocusing quadrupole (ie, four sets total). Express your answers in terms of (as appropriate) β_{max}, β_{min}, F, and L. (Hint: invoke appropriate symmetry arguments to simplify the problem.) (4 points)

We'll calculate the Twiss parameters before and after each quadrupole, as inticated by locations "1" through "4" below:



You showed in HW 4.1 that a thin quadrupole transforms the lattice function as

$$\alpha_1 = \alpha_0 + \beta_0 \frac{1}{f}$$

$$\beta_1 = \beta_0$$

$$\gamma_1 = \gamma_0 + 2\frac{\alpha_0}{f} + \frac{\beta_0}{f^2}$$

We know from the calculations you did for Lab 4 (or the first problem in last year's final) that the Twiss parameters at the centers of the focusing and defocusing quads are given by

$$\begin{pmatrix} \alpha_0 \\ \beta_0 \\ \gamma_0 \end{pmatrix} = \begin{pmatrix} 0 \\ \beta_{max} \\ \left(\frac{1}{\beta_{max}}\right) \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ \beta_{min} \\ \left(\frac{1}{\beta_{min}}\right) \end{pmatrix}$$

respectively, so we can quickly the lattice functions at locations 1 as

$$\alpha_1 = \boxed{\frac{\beta_{max}}{2F}}$$

$$\beta_1 = \boxed{\beta_{max}}$$

$$\gamma_1 = \boxed{\frac{1}{\beta_{max}} + \frac{\beta_{max}}{4F^2}}$$

We can get the lattice functions just after the defucusing quadrupole (point 3) by substituting β_{min} for β_{max} and changing the sign of F

$$\alpha_3 = \boxed{-\frac{\beta_{min}}{2F}}$$

$$\beta_3 = \boxed{\beta_{min}}$$

$$\gamma_3 = \boxed{\frac{1}{\beta_{min}} + \frac{\beta_{min}}{4F^2}}$$

There are two ways to solve for the lattice parameters at locations 2 and 4. The easiest is to invoke a symmetry argument. If we reverse the direction of the FODO cell, it looks exactly the same. Since γ and β are related to the distributions of the particles, they will clearly be the same before and after each quadrupole. On the other hand $\alpha = -1/2 d\beta/ds$ is a derivative, so to maintain the overall symmetry of the problem, it must have an opposite sign on each side of the the quadrupoles, so

$$\alpha_2 = -\alpha_3 = \boxed{-\frac{\beta_{min}}{2F}}$$

$$\beta_2 = \beta_3 = \boxed{\beta_{min}}$$

$$\gamma_2 = \gamma_3 = \boxed{\frac{1}{\beta_{min}} + \frac{\beta_{min}}{4F^2}}$$

and

$$\alpha_4 = -\alpha_1 = \boxed{-\frac{\beta_{max}}{2F}}$$

$$\beta_4 = \beta_1 = \boxed{\beta_{max}}$$

$$\gamma_4 = \gamma_1 = \boxed{\frac{1}{\beta_{max}} + \frac{\beta_{max}}{4F^2}}$$

If you don't like the symmetry argument, then you can plug these back into the original equations and solve, as

$$\beta_0 = \beta_{max} = \beta_4 \rightarrow \beta_4 = \boxed{\beta_{max}}$$

$$\alpha_0 = 0 = \alpha_4 + \beta_4/(2F) \rightarrow \alpha_4 = \boxed{-\frac{\beta_{max}}{2F}}$$

$$\gamma_0 = \frac{1}{\beta_{max}} = \gamma_4 + 2\frac{\alpha_4}{(2F)} + \frac{\beta_4}{(2F)^2} = \gamma_4 - \frac{\beta_{max}}{2F^2} + \frac{\beta_{max}}{4F^2} = \gamma_4 - \frac{\beta_{max}}{4F^2} \rightarrow \gamma_4 = \boxed{\frac{1}{\beta_{max}} + \frac{\beta_{max}}{4F^2}}$$

You can then plug in the lattice functions at the center of the defocusing quadrupole and solve for location 2 as

$$\beta_{defocus} = \beta_{min} = \beta_2 \rightarrow \beta_2 = \boxed{\beta_{min}}$$

$$\alpha_{defocus} = 0 = \alpha_2 - \beta_2/(2F) \rightarrow \alpha_3 = \boxed{+\frac{\beta_{min}}{2F}}$$

$$\gamma_{defocus} = \frac{1}{\beta_{min}} = \gamma_2 - 2\frac{\alpha_2}{(2F)} + \frac{\beta_2}{(2F)^2} = \gamma_2 - \frac{\beta_{min}}{2F^2} + \frac{\beta_{min}}{4F^2} = \gamma_2 - \frac{\beta_{min}}{4F^2} \rightarrow \gamma_2 = \boxed{\frac{1}{\beta_{min}} + \frac{\beta_{min}}{4F^2}}$$

Note! The fact that the γ function doesn't change over a drift implies that

$$\gamma_2 = \gamma_1 = \gamma_4 = \gamma_3 \rightarrow \frac{1}{\beta_{max}} + \frac{\beta_{max}}{4F^2} = \frac{1}{\beta_{min}} + \frac{\beta_{min}}{4F^2}$$

This can in fact be proven with a little bit of algebra

b. The quadrupoles are placed L=10m apart, and the desired phase advance per cell is 103°. What is the required focal length F? [m] (2 points)

From our discussion in class and from the calculations you did for Lab4, we have that

$$\sin\frac{\mu}{2} = \frac{L}{2F} \rightarrow F = \frac{L}{2\sin\frac{\mu}{2}} = \frac{(10)}{2\sin\left(\frac{103^\circ}{2}\right)} = \boxed{6.39 \text{ m}}$$

c. What are the values of β_{max} and β_{min}? [m] (2 points)

From our discussion in class and from the calculations you did for Lab4, we have that

$$\beta max, min = 2L\frac{1 \pm \sin\frac{\mu}{2}}{\sin\mu} = 2(10)\frac{1 \pm \sin\left(\frac{103^\circ}{2}\right)}{\sin(103^\circ)} \rightarrow \boxed{\beta_{max} = 36.59 \text{ m}, \beta_{min} = 4.46 \text{ m}}$$

d. If I build a ring out of N=58 of these cells, what are the circumference C [m] and the tune ν [number]? (2 points)

Each cell is $2L$ long, so the total circumference is

$$C = 2LN_{cell} = 2(10)(58) = \boxed{1160 \text{ m}}$$

The tune will be given by

$$\nu = \frac{\mu}{2\pi} = \frac{(58)(103^\circ)}{(360^\circ)} = \boxed{16.594}$$

e. As you learned when you studied coupling, having exactly equal tunes in both planes is not actually a good idea, If I modify each cell by uniformly increasing the magnetic gradient of *all 58* of the focusing quadrupoles by 0.1%, what will be the approximate total change in the tunes ν_x and ν_y in the x and the y planes, respectively? [numbers] (Hint: think of the change in gradient as the *addition* of a small quadrupole right next to the existing one, and be careful with your signs.) (4 points)

Increasing the gradient of the focusing quads by a small fractional amount κ is equivalent to adding a quad right next to it with an effective strength given by

$$\frac{1}{f_{eff}} = \kappa \frac{1}{F}$$

We learned in class (p. 7 of the "Imperfections" lecture) that the tuneshift caused by a small quadrupole term is

$$\Delta\nu = \frac{1}{4\pi}\frac{\beta}{f}$$

The tuneshifts in each cell will add, so the total tuneshift in the x plane will be

$$\Delta\nu_x = N_{cell}\frac{1}{4\pi}\beta_{max}\kappa\frac{1}{F} = (58)\frac{1}{4\pi}(36.6)(.001)\frac{1}{(6.4)} = \boxed{.0264}$$

In the y plane, $\beta_{max} \to \beta_{min}$ and $F \to -F$, so

$$\Delta\nu_y = N_{cell}\frac{1}{4\pi}\beta_{min}\kappa\frac{1}{-F} = (58)\frac{1}{4\pi}(4.46)(.001)\frac{1}{(-6.4)} = \boxed{-.00322}$$

f. If my ring has anomalous magnetic errors up to and including sextupole terms, what values of the fractional tune could cause resonant instabilities? (1 point)

This question appeared *verbatim* on the 2014 final exam.

We showed ("Floquet coordinates and resonances", page 10) that quadrupoles can lead to resonances at the whole and half integer tunes and that sextupoles can lead to resonances at third integer tunes, so we should avoid tunes with a fractional component of

$$\boxed{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \text{ and } 1}$$

g. Based on your answers to the previous two questions, what would be the smallest magnitude change in the gradients of the 58 focusing magnets that could subject the beam to a resonant instability? [percentage] (be sure to look at both signs and both planes) (3 points)

We showed in part (c) that the unperturbed tune is ν_x=ν_y=16.594. The nearest resonance in both planes will be $\nu = 16\frac{2}{3}$, so a tuneshift of $\Delta\nu = (16\frac{2}{3} - 16.594) = +.07222$ wil put the machine onto a third integer resonance. Because the β function is larger in the x plane, it will take a smaller current to reach the resonance. From the previous question, we have

$$\Delta\nu_x = N_{cell}\frac{1}{4\pi}\beta_{max}\kappa\frac{1}{F}$$

Solving for κ, we have

$$\kappa = \Delta\nu\frac{4\pi F}{N_{cell}\beta_{max}} = (.07222)\frac{4\pi(6.4)}{(58)(36.6)} = .00273 = \boxed{.273\%}$$

Problem 2 (24 points total)

Assume that the unperturbed lattice described above is designed to accelerate protons from a kinetic energy of K_{min}=5 GeV to a kinetic energy of K_{max}=50 GeV. The injected beam has a normalized RMS emittance of 1 μm in both planes. The quadrupoles are each 2 m long.

a. Calculate the momentum p [GeV/c], relativistic β, period τ [μsec], and beam rigidity $(B\rho)$ [T-m] at the minimum and maximum energies. (6 points)

We can build a table with the parameters we will need now and later. The requested values are shown boxed.

Value	Formula	Injection	Extraction
K [GeV]	-	5	50
m [GeV/c^2]	-	.938	.938
E_s [GeV]	$K + mc^2$	5.938	50.938
p [GeV/c]	$\sqrt{E_s^2 - (mc^2)^2}/c$	$\boxed{5.86}$	$\boxed{50.93}$
β	pc/E_s	$\boxed{0.987}$	$\boxed{0.999}$
γ	$E_s/(mc^2)$	6.33	54.30
$\beta\gamma$	$p/(mc^2)$	6.25	54.30
$(B\rho)$ [T-m]	p[GeV]/.300	$\boxed{19.54}$	$\boxed{169.76}$
C [m]	$2LN_{cell}$	1160	1160
τ μsec	$C/(\beta c)$	$\boxed{3.92}$	$\boxed{3.92}$

b. What is the maximum magnetic gradient needed in the quadrupoles? [T/m] (2 points)

We calculated the required focal length in 1b. The relations ship between the focal length and magnet parameters is

$$\frac{1}{F} = \frac{B'L}{(B\rho)}$$

The highest gradient will be needed at the highest energy, so we can solve for the required magnetic gradient with

$$B' = \frac{(B\rho)}{FL} = \frac{(169.76)}{(6.4)(2)} = \boxed{13.23 \text{ T/m}}$$

c. Assuming the injected beam is properly matched, what are the maximum and minimum beam sizes $\sigma_{x,max}$ and $\sigma_{x,min}$ at the minimum energy? [mm] (4 points)

The minimum and maximum beam sizes are calculated by the minimum and maximum beta functions and the normalized emittance

$$\sigma_{x,min} = \sqrt{\frac{\epsilon_N \beta_{min}}{\beta\gamma}} = \sqrt{\frac{(.000001)(4.46)}{(6.25)}} = .000845 \text{ m} = \boxed{.845 \text{ mm}}$$

$$\sigma_{x,max} = \sqrt{\frac{\epsilon_N \beta_{max}}{\beta\gamma}} = \sqrt{\frac{(.000001)(35.58)}{(6.25)}} = .00242 \text{ m} = \boxed{2.42 \text{ mm}}$$

d. Assuming the injected beam is properly matched, what are the maximum and minimum angular distributions $\sigma_{x',max}$ and $\sigma_{x',min}$ at the minimum energy? [radians] (4 points)

The maximum $\sigma_{x'}$ will occur where σ_x is minumum, at the center of the defocusing quads, and the minimum $\sigma_{x'}$ will occur at the center of the focusing quads, so

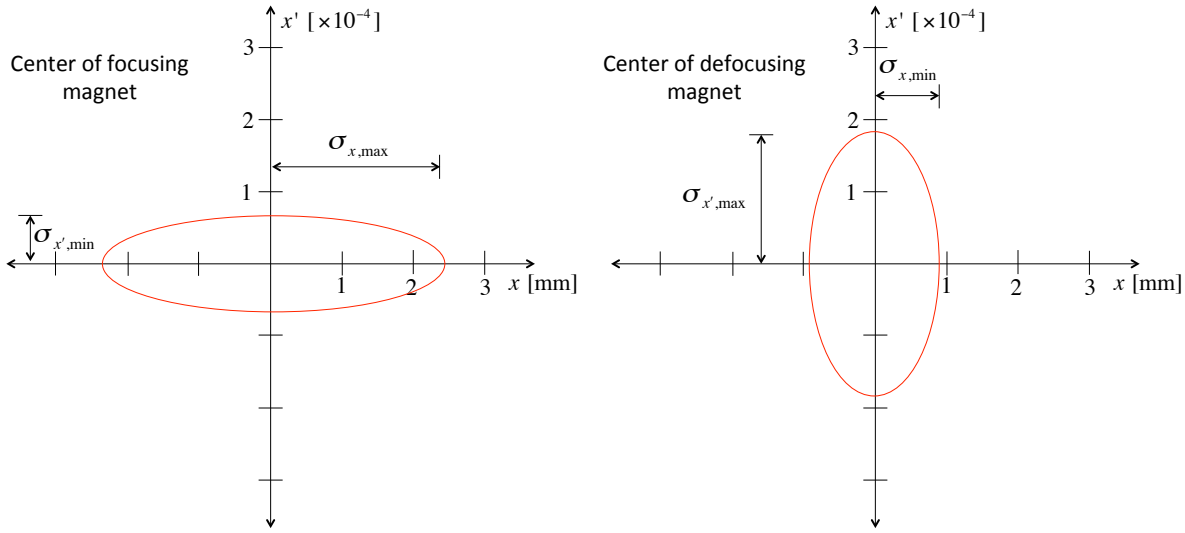
$$\sigma_{x',max} = \sqrt{\frac{\epsilon_N \gamma_{max}}{\beta\gamma}} = \sqrt{\frac{\epsilon_N}{\beta_{min}\beta\gamma}} = \sqrt{\frac{(.000001)}{(4.46)(6.25)}} = \boxed{1.89 \times 10^{-4}}$$

and will occur at the center of the focusing quads, so

$$\sigma_{x',min} = \sqrt{\frac{\epsilon_N \gamma_{min}}{\beta\gamma}} = \sqrt{\frac{\epsilon_N}{\beta_{max}\beta\gamma}} = \sqrt{\frac{(.000001)}{(36.6)(6.25)}} = \boxed{6.61 \times 10^{-5}}$$

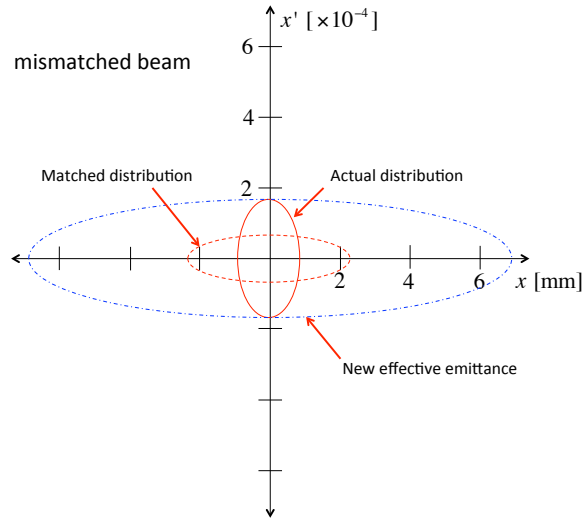
e. Based on your previous two answers, sketch the phase space distributions in the x plane at the center of the focusing and defocusing quads at injection (minimum energy). In each plot, clearly indicate the RMS extrema $\sigma_{x,max}$, $\sigma_{x,min}$, $\sigma_{x',max}$ and $\sigma_{x',min}$ that you calculated above. (The plots need not be terribly precise, but should be drawn to the same scale such that the relative sizes of the key features are qualitatively correct). (4 points)

The distribution at the center of the focusing magnet will have $\sigma_x = \sigma_{x,max}$ and $\sigma_{x'} = \sigma_{x',min}$ and the distribution at the center of the defocusing magnet will have $\sigma_x = \sigma_{x,min}$ and $\sigma_{x'} = \sigma_{x',max}$ as illustrated below:



f. Now assume that the injected beam has the correct emittance, but is mismatched, such that the phase space distribution at the center of the first *focusing* magnet it encounters looks instead like the distribution for the *defocusing* magnet in the previous question. In this case, what is the new effective (diluted) emittance of this mismatched beam? [μm] (Hint: this question is really easy. If you're doing a lot of work, you're doing it wrong.). (4 points)

As we discussed in class (see page 2 of "Matching and insertions"), the individual particles of the injected beam will follow the same trajector that a matched particle would follow, leading to the new effective emittance illustrated below



There are several ways to calculate the effective emittance. We see that the new effective emittance will be defined be the size of $\sigma_{x'}$. In the mismatched beam, this is

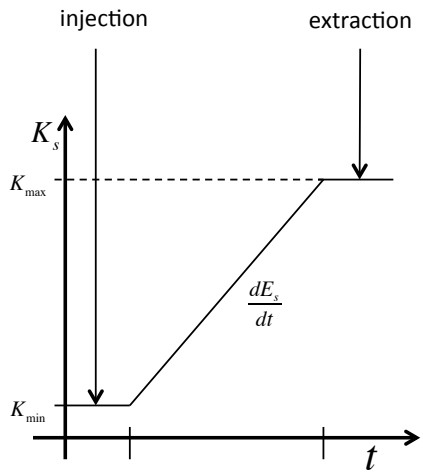
$$\sigma'_{x'} = \sigma_{x',max} = \sqrt{\frac{\epsilon_N}{\beta_{min}\beta\gamma}}$$

But we can also express this as an effective emittance and equate the two

$$\sigma'_{x'} = \sqrt{\frac{\epsilon_{eff,N}}{\beta_{max}\beta\gamma}} = \sqrt{\frac{\epsilon_N}{\beta_{min}\beta\gamma}} \Longrightarrow \epsilon_{eff,N}\frac{\beta_{max}}{\beta_{min}} = (1)\frac{(36.59)}{(4.46)} = \boxed{8.2\ \mu\text{m}}$$

Problem 3 (28 points total)

In this problem, we will consider the RF system for the synchrotron described above. The RF system operates at a harmonic h=188 and the transition gamma is γ_t=15. When the beam is injected, the synchroton is not accelerating. Once the protons have been injected, they are accelerated from the kinetic energy K_{min}=5 GeV to K_{max}=50 GeV and then extracted, as illustrated below



The injected beam is transferred from another accelerator, where it was bunched at the same RF frequency f_{inj}. Each bunch in the injected beam has an RMS energy spread of σ_E/E of 0.1% and an RMS time distribution of σ_t=2 ns.

a. What are the RF frequencies f_{inj} and f_{ext} at injection and extraction? [MHz]. (3 points)

We calculated β_{inj} and β_{ext} in problem 2a above, so

$$f_{inj} = \frac{h}{\tau_{inj}} = \frac{h\beta c}{C} = \frac{(188)(.987)(3\times 10^8)}{(1160)} = 48.0\times 10^6 = \boxed{48.0 \text{ MHz}}$$

$$f_{ext} = \frac{h}{\tau_{ext}} = \frac{h\beta c}{C} = \frac{(188)(1.0)(3\times 10^8)}{(1160)} = 48.6\times 10^6 = \boxed{48.6 \text{ MHz}}$$

b. What are the slip factors η_{inj} and η_{ext} at injection and extraction? (3 points)

We showed on "Off Momentum Particles", slide 7 that

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \rightarrow \eta_{inj} = \frac{1}{(15)^2} - \frac{1}{(6.33)^2} = \boxed{-.021}, \eta_{ext} = \frac{1}{(15)^2} - \frac{1}{(54.3)^2} = \boxed{.0041}$$

c. What is the RMS longitudinal emittance ϵ_L of the injected beam? [eV-s] (2 points)

The normalized longitudinal emittance is given be the product of the time disribution and energy distribution

$$\epsilon_L = \sigma_E\sigma_t = E_s\frac{\Delta E}{E}\sigma_t = (5.938\times 10^9)(.001)(2\times 10^{-9}) = \boxed{.012 \ \text{eV}-\text{s}}$$

d. If I want to correctly match my RF to the longitudinal bunch shape of the injected beam, what value of peak RF voltage V_0 do I need? [MV] (4 points)

The shape of the bunch is described by the longitudinal beta function (see "Longitudinal Motion, p. 8 and 9")

$$\beta_L = \frac{\sigma_t}{\sigma_e} = \frac{(2 \times 10^{-9})}{(5.938 \times 10^9)(.001)} = 3.37 \times 10^{-16} \text{ s/eV}$$

If it is correctly matched to the RF system, then it is related to the RF system by

$$\beta_L = \sqrt{-\frac{\tau\eta}{eV_0\omega_{RF}\beta^2\cos\phi_s}} = \sqrt{-\frac{\tau^2\eta}{2\pi eV_0 h\beta^2\cos\phi_s}}$$

Putting in the non-accelerating case ($\cos\phi_s = \pm 1$) and solving for V_0, we get

$$V_0 = \frac{1}{e}\frac{\tau^2|\eta|}{2\pi\beta_L^2 hE_s\beta^2} = \frac{1}{e}\frac{(3.9 \times 10^{-6})^2(.021)}{2\pi(3.37 \times 10^{-16})^2(188)(5.938)(.987)^2} = 405324 = \boxed{.41 \text{ MV}}$$

e. I advance the synchronous phase angle to ϕ_s=60°. What is the initial acceleration of the beam dE_s/dt? [GeV/s] (2 points)

The energy ramp rate is given by the energy gained per turn over the period

$$\frac{dE_s}{dt} = \frac{V_0\sin\phi}{\tau} = \frac{(.41 \times 10^6)(\sin 60°)}{(3.9 \times 10^{-6})} = 89.6 \times 10^9 = \boxed{89.6 \text{ GeV/s}}$$

f. I adiabatically accelerate the beam to K_{max}, and then adiabatically stop accelerating. If my peak voltage is still the V_0 I calculated above, what are the values for the RMS values σ_E [MeV] and σ_t [ns] at this point? (4 points)

If we accelerated the beam adiabatically, then the longidinal emittance which we calculated in part b is conserved, so we just need to calculate the new longidunal beta function

$$\beta_L = \sqrt{-\frac{\tau\eta}{eV_0\omega_{RF}\beta^2\cos\phi_s}} = \sqrt{-\frac{\tau^2\eta}{2\pi eV_0 h\beta^2\cos\phi_s}} = \frac{(3.9 \times 10^{-6})^2(0.0041)}{2\pi(.41 \times 10^6)(\sin 60°)(188)(1)^2} = 5.0 \times 10^{-17}$$

We can then use this and the longitudinal emittance to calculate the new energy and time distributions

$$\sigma_E = \sqrt{\frac{\epsilon_L}{\beta_L}} = \sqrt{\frac{(.012)}{(5.0 \times 10^{-17})}} = 15.4 \times 10^6 = \boxed{15.4 \text{ MeV}}$$

$$\sigma_t = \sqrt{\epsilon_L\beta_L} = \sqrt{(.012)(5.0 \times 10^{-17})} = 7.72 \times 10^{-10} = \boxed{.77 \text{ ns}}$$

g. I wish to do a bunch rotation to decrease the time distribution slightly prior to extraction. I quickly (non-adabatically) increase the RF voltage to $V_0' = 4 * V_0$. How many revolution periods of the machine do I then need to wait before extracting the beam in order to achieve the minimum RMS time distribution σ_t? [turns] (4 points)

As discussed in the "Longitudinal Motion 2" lecture, we'll want to let the bunch rotate for a quarter of a synchrotron period. We can calculate the synchrotron tune from ("Longitudinal Motion 1", page 6) using the the new voltage $V = 4V_0$ and $\cos\phi_s = -1$

$$\nu_s = \frac{1}{2\pi}\sqrt{-\frac{eV_0'\omega_{RF}\tau\eta}{E_s\beta^2}\cos\phi_s} = \frac{1}{2\pi}\sqrt{-\frac{eV_0'2\pi h\eta}{E_s\beta^2}} = \sqrt{\frac{eV_0'h\eta}{2\pi E_s\beta^2}} = \sqrt{\frac{(4\times .41\times 10^6)(188)(.0041)}{2\pi(50.938\times 10^9)(1.)^2}} = .00198$$

We then want to wait 1/4 of a synchrotron cycle to extract at with the narrowest σ_t, or

$$n_{rot} = \frac{1}{4}\frac{1}{\nu_s} = \frac{1}{4(.0198)} \approx \boxed{126 \text{ turns}}$$

h. If I extract the beam at this optimum time, what is the RMS of the time distribution σ_t of each extracted bunch? [ns] (Hint: this is not hard, but you have to think a bit. Consider the relative sizes of the dimensions of the bunch to what they would be for a properly matched bunch right after the voltage is increased, and then think about what this implies after the rotation.). (6 points)

Changing the voltage (and nothing else) will change the longitudinal beta, which is given by

$$\beta_L = \sqrt{-\frac{\tau\eta}{eV_0\omega_{RF}\beta^2\cos\phi_s}} \propto \frac{1}{\sqrt{V_0}}$$

so quadrupling the voltage will reduce the longitudinal beta to

$$\beta_L' = \frac{1}{\sqrt{4}}\beta_L = \frac{1}{2}\beta_L$$

The energy and time distributions of a matched bunch are given by

$$\sigma_E' = \sqrt{\frac{\epsilon_L}{\beta_L}} \propto\propto \frac{1}{\sqrt{\beta_L}} \propto V_0^{1/4}$$

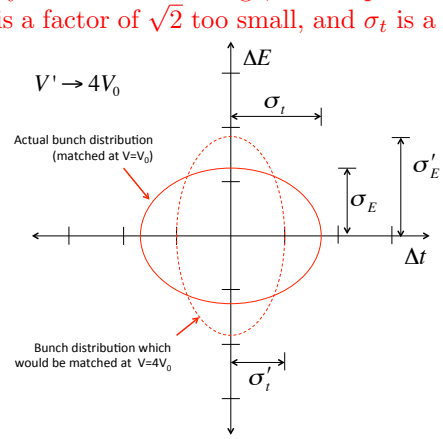
and

$$\sigma_t' = \sqrt{\epsilon_L\beta_L} \propto \sqrt{beta_L} \propto \frac{1}{V_0^{1/4}}$$

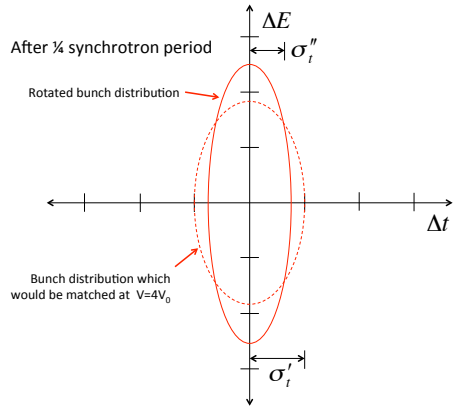
So if we had *adiabatically* increased the voltage by a factor of 4, then the energy and time distributions would have changed to

$$\sigma_E' = 4^{1/4}\sigma_E = \sqrt{2}\sigma_E \quad \text{and} \quad \sigma_t' = \frac{1}{4^{1/4}}\sigma_t = \frac{1}{\sqrt{2}}\sigma_t$$

However, we non-adiabtically increase the voltage, so compared to a matched bunch of the same longitudinal emittance, σ_E is a factor of $\sqrt{2}$ too small, and σ_t is a factor of $\sqrt{2}$, as illustrated below



The individual particles will follow trajectory of a matched bunch, so after 1/4 of a synchrotron period, the situation will reverse, with σ_E being a factor of $\sqrt{2}$ too large and σ_t being a factor of $\sqrt{2}$ too small, as shown below



So we can see that the new time distribution will be

$$\sigma_t'' = \frac{\sigma_t'}{\sqrt{2}} = \frac{\sigma_t/sqrt2}{\sqrt{2}} = \frac{\sigma_t}{2} = \frac{(.77)}{2} = \boxed{.34 \text{ ns}}$$