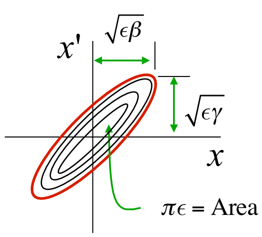
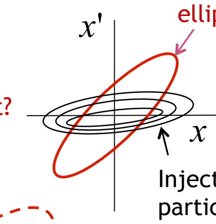
Insertions and Matching

Mismatch and Emittance Dilution

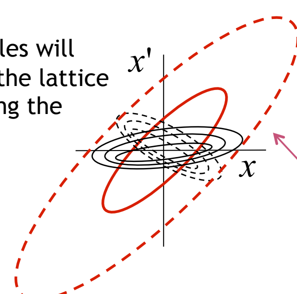
- In our previous discussion, we implicitly assumed that the distribution of particles in phase space followed the ellipse defined by the lattice function



…but there's no guarantee
What happens if this it's not?



- Once injected, these particles will follow the path defined by the lattice ellipse, effectively increasing the emittance

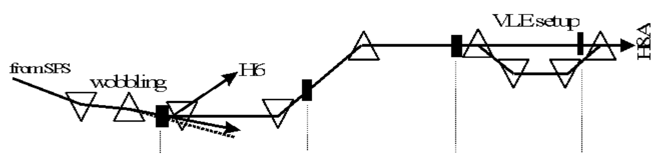


Beam Lines

- In our definition and derivation of the lattice function, a closed path through a periodic system. This definition doesn't exist for a beam line, but once we know the lattice functions at one point, we know how to propagate the lattice function down the beam line.

$$\overbrace{\qquad\qquad\qquad\qquad}^{\mathbf{M}(out,in)}$$

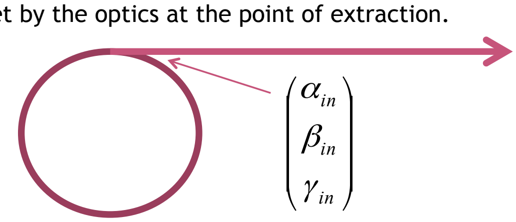
$$\begin{pmatrix}\alpha_{in}\\ \beta_{in}\\ \gamma_{in}\end{pmatrix} \qquad \begin{pmatrix}\alpha_{out}\\ \beta_{out}\\ \gamma_{out}\end{pmatrix}$$



$$\begin{pmatrix}\alpha_{out}\\ \beta_{out}\\ \gamma_{out}\end{pmatrix} = \begin{pmatrix} (m_{11}m_{22}+m_{12}m_{21}) & (-m_{11}m_{21}) & (-m_{12}m_{22}) \\ (-2m_{11}m_{12}) & (m_{11}^2) & (m_{12}^2) \\ (-2m_{21}m_{22}) & (m_{21}^2) & (m_{22}^2) \end{pmatrix} \begin{pmatrix}\alpha_{in}\\ \beta_{in}\\ \gamma_{in}\end{pmatrix}$$

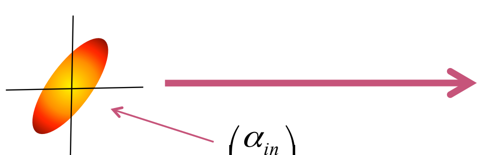
Establishing Initial Conditions

- When extracting beam from a ring, the initial optics of the beam line are set by the optics at the point of extraction.



$$\begin{pmatrix}\alpha_{in}\\ \beta_{in}\\ \gamma_{in}\end{pmatrix}$$

- For particles from a source, the initial lattice functions can be defined by the distribution of the particles out of the source



$$\begin{pmatrix}\alpha_{in}\\ \beta_{in}\\ \gamma_{in}\end{pmatrix}$$

Insertions

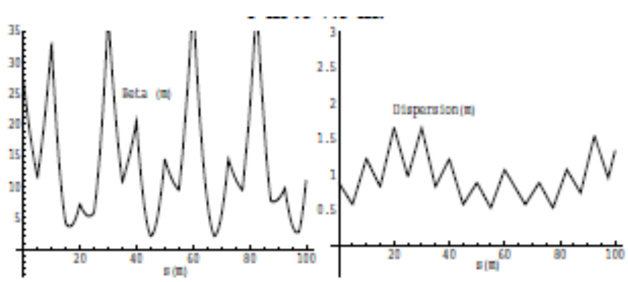
- So far, we've talked about nice, periodic lattice, but that may not be all that useful in the real world. In particular, we generally want
  - Locations for injection of extraction.
  - "Straight" sections for RF, instrumentation, etc
  - Low beta points for collisions
- Since we generally think of these as taking the place of things in our lattice, we call them "insertions"

FODO	FODO	FODO	Insertion	FODO	FODO	FODO

Match lattice functions

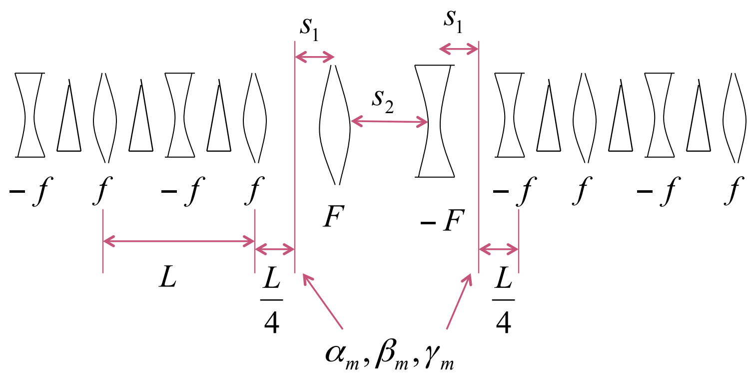
Mismatch and Beta Beating

- Simply modifying a section of the lattice without matching will result in a distortion of the lattice functions around the ring (sometimes called "beta" beating)
- Here's an example of increasing the drift space in one FODO cell from 5 to 7.5 m



Collins Insertion

- A Collins Insertion is a way of using two quads to put a straight section into a FODO lattice



- Where s_2 is the usable straight region

- Require that the lattice functions at both ends of the insertion match the regular lattice functions at those point

$$M = \begin{pmatrix} 1 & s_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{F} & 1 \end{pmatrix} \begin{pmatrix} 1 & s_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix} \begin{pmatrix} 1 & s_1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\mu_I + \alpha_m \sin\mu_I & \beta_m \sin\mu_I \\ -\gamma_m \sin\mu_I & \cos\mu_I - \alpha_m \sin\mu_I \end{pmatrix}$$

Where μ_I is a free parameter

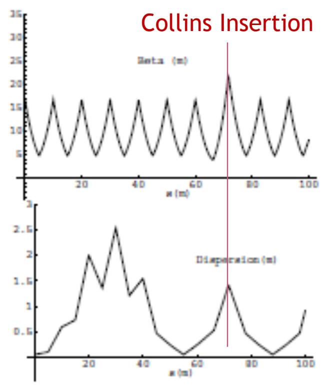
- After a bit of algebra

$$s_1 = \frac{\tan\frac{\mu_I}{2}}{\gamma}; s_2 = \frac{\alpha^2 \sin\mu_I}{\gamma}; F = -\frac{\alpha}{\gamma}$$

- Maximize s_2 with $\mu_I = \pi/2$, α max (which is why we locate it *L*/2 from quad)
- Works in both planes if $\alpha_x = -\alpha_y$ (true for simple FODO)

Dispersion Suppression

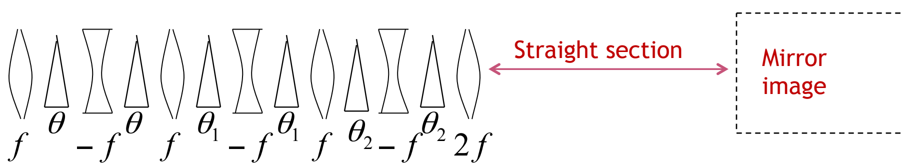
- The problem with the Collins insertion is that it does *not* match dispersion, so just sticking it in the lattice will lead to distortions in the dispersion



- This is typically dealt with by suppressing the dispersion entirely in the region of the insertion.

Dispersion Suppression (cont'd)

- On common technique is called the "missing magnet" scheme, in which the FODO cells on either side of the straight section are operated with two different bending dipoles and a half-strength quad



- Recall that the dispersion matrix for a FODO half cell is (lecture 6)

$$\begin{pmatrix} D \\ D' \\ 1 \end{pmatrix} = \mathbf{M} \begin{pmatrix} D_0 \\ D_0' \\ 1 \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 0 \\ -\frac{1}{2f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & \frac{L\theta}{2} \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ \frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & \frac{L\theta}{2} \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ -\frac{1}{2f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{L^2}{2f^2} & 2L\left(1+\frac{L}{2f}\right) & 2L\theta\left(1+\frac{L}{4f}\right) \\ -\frac{L}{2f^2}+\frac{L^2}{4f^3} & 1-\frac{L^2}{2f^2} & 2\theta\left(1-\frac{L}{4f}-\frac{L^2}{8f^2}\right) \\ 0 & 0 & 1 \end{pmatrix}$$

- So we solve for

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \mathbf{M}(\theta = \theta_2)\mathbf{M}(\theta = \theta_1)\begin{pmatrix} D_m \\ D'_m \\ 1 \end{pmatrix}$$

- Where D_m and D'_m are the dispersion functions at the end of a normal cell (for a simple lattice, D'_m=0)
- We get the surprisingly simple result

$$\theta_1 = \theta\left(1 - \frac{1}{4\sin^2\frac{\mu}{2}}\right); \theta_2 = \theta\frac{1}{4\sin^2\frac{\mu}{2}}$$

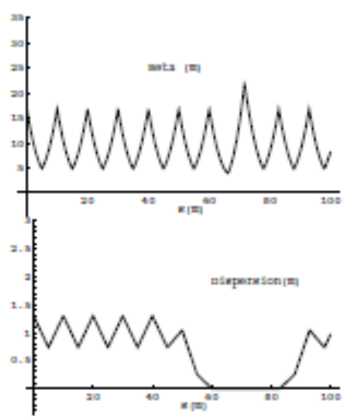
- Note that if μ=60°

$$\theta_1 = 0 \qquad \theta_2 = \theta$$

- So the cell next to the insertion is normal, and the next one has no magnets, hence the name "missing magnet".

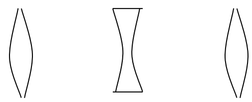
Combining Insertions

- Because the Collins Insertion has no bend magnets, it cannot generate dispersion if there is none there to begin with, so if we put a Collins Insertion inside of a dispersion suppressor, we match both dispersion and the lattice functions.



Focusing Triplet

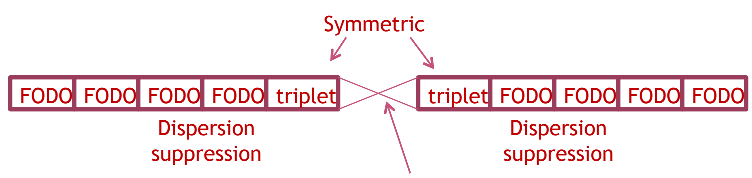
- In experimental applications, we will often want to focus beam down to a waist (minimum β) in both planes. In general, we can accomplish this with a triplet of quadrupoles (you'll learn more about this in the computer lab).



- Such triplets are a workhorse in beam lines, and you'll see them wherever you want to focus beam down to a point.
- The solution, starting with a arbitrary lattice functions, is not trivial and in general these problems are solved numerically.

Low β Insertions

- In a collider, we will want to focus the beam in both planes as small as possible.
- This can be done with a symmetric pair of focusing triplets, matched to the lattice functions (dispersion suppression is assumed)



$\beta = \beta^*; \alpha = 0$

- Recall that in a drift, β evolves as

$$\beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2 = \beta^* + \frac{s^2}{\beta^*}$$

Where *s* is measured from the location of the waist

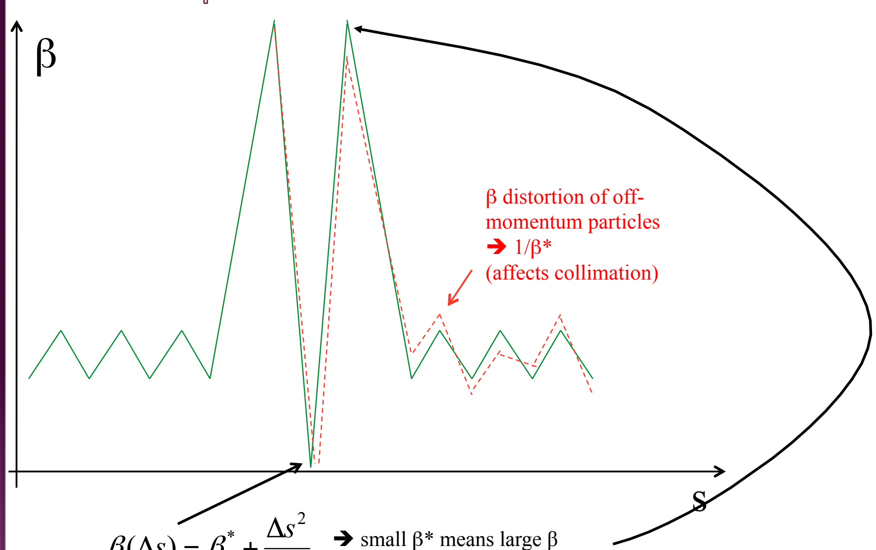
Phase Advance of a Low Beta Insertion

- We can calculate the phase advance of the insertion as

$$\Delta\psi = \int_{-L/2}^{L/2} \frac{ds}{\beta} = \frac{1}{\beta^*}\int_{-L/2}^{L/2} \frac{ds}{\left(1+\left(\frac{s}{\beta^*}\right)^2\right)} = 2\tan^{-1}\left(\frac{L}{\beta^*}\right)$$

- For L>>β*, this is about π, which guarantees that all the lattice parameters will match except dispersion (and we've suppressed that).
- This means that each low beta insertion will increase the tune by about 1/2

Limits to β*

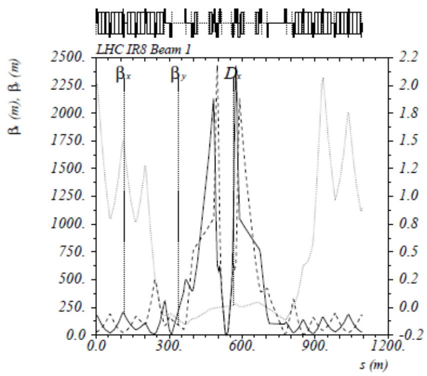


$$\beta(\Delta s) = \beta^* + \frac{\Delta s^2}{\beta^*}$$

➔ small β* means large β (aperture) at focusing triplet

Example: The Case for New LHC Quadupoles

- Proposal to increase luminosity: β*=55 cm ➔ β*=10 cm



Existing quads
- 70 mm aperture
- 200 T/m gradient

Proposed for upgrade
- At least 120 mm aperture
- 200 T/m gradient
- Field 70% higher at pole face

Beyond the limit of NbTi
Must go to Nb_3Sn

- Need bigger quads to go to smaller β*