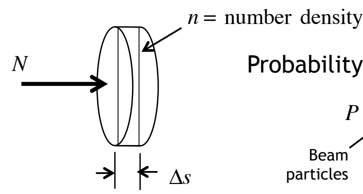
Beam Loss and Emittance Growth Mechanisms

Eric Prebys, FNAL

Beam Loss and Emittance Growth

- Lifetime
  - Residual gas interaction
  - Touscheck effect
  - Quantum lifetimes (electron machines)
  - Beam-beam collisions
- Emittance growth
  - Residual gas interactions
  - Intrabeam scattering
  - Random noise sources

Residual Gas Interactions



Probability of interaction

$$P = Nn\sigma\Delta s$$

N: Beam particles; n: Number density; σ: Cross section; Δs: Path length

Assume all particles lost, then

$$\frac{dN}{dt} = -Nn\frac{ds}{dt}\sigma = -Nnv\sigma = -\frac{N}{\tau}$$

$$\rightarrow \boxed{\tau = \frac{1}{nv\sigma}}$$

Coulomb Scattering

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{2\pi\theta d\theta} = \frac{4Z^2 r_0^2}{(\theta+\theta_1)^2}\left(\frac{m_e c}{p}\right)^2;\ \ \theta_1 = \alpha Z^{1/3}\left(\frac{m_e c}{p}\right)$$

If the particle is scattered at an angle θ and a point where the lattice function is β, then the average scattering amplitude at a limiting aperture will be.

$$x_{rms} = \theta\frac{\sqrt{\beta\beta_{lim}}}{\sqrt{2}} \rightarrow \theta_{lim} = \frac{\sqrt{2}b}{\sqrt{\beta\beta_{lim}}}$$

b: Limiting aperture

So the coulomb loss cross-section will be

$$\sigma_{coulomb} = 2\pi\int_{\theta_{lim}}^{\pi}\frac{d\sigma}{d\Omega}\theta\, d\theta = \frac{4\pi Z^2 r_0^2}{\theta_{lim}^2}\left(\frac{m_e c}{p}\right)^2$$

$$= \frac{\beta_{lim}\langle\beta\rangle}{b^2}2\pi Z^2 r_0^2\left(\frac{m_e c}{p}\right)^2$$

Example: Nitrogen (Z=7)

$$b = 30 \text{ mm}$$
$$\beta_{lim} = 15 \text{ m}$$
$$\text{p=5 GeV/c}$$
$$\longrightarrow \sigma_{coulomb} = 1.3\times10^{-25}\, cm^2$$
$$= .13 \text{ barn}$$

Bremsstrahlung (electrons only)

The cross section of beam lost to bremsstrahlung is

$$\frac{d\sigma}{du} \approx \frac{16\alpha r_0^2}{3} Z(Z+1)\ln\left(\frac{184}{Z^{1/3}}\right)\frac{1-u+.75u^2}{u};\ \ u \equiv \frac{\Delta E}{E}$$

If ΔE_a is the energy aperture of the machine, then

$$\sigma_{brem} = \int_{u_a}^{\infty} \frac{d\sigma}{du} du \approx \frac{16\alpha r_0^2}{3} Z(Z+1)\ln\left(\frac{184}{Z^{1/3}}\right)\left[\ln\frac{1}{u_a} - \frac{5}{8}\right]$$

$$\frac{\Delta E_a}{E} \rightarrow u_a$$

Example: Nitrogen (Z=7)

$$u_a = .003 \rightarrow \sigma_{brem} = 4 \text{ barn} >> \sigma_{coulomb}$$

Proton Beam Lifetimes

Bremsstrahlung is not (yet) an issue for protons, but nuclear interaction are. There is no simple formula, but they are typically a fraction of a barn and have a very weak energy dependence. The nuclear cross section for Nitrogen is .4 barns.

Thus, the total cross section for proton loss is. $\sigma = \sigma_{coulomb} + \sigma_{nuclear}$

$$\frac{1}{\tau} = nv\sigma$$

For and ideal gas

$$n_{mol} = \frac{P}{k_b T} = 9.66\times10^{24}\ \frac{P[\text{nTorr}]}{T[\text{K}]}$$

Molecular density (n_{mol}), Pressure (P), Temperature (T), Boltzmann Constant (k_b)

For for a diatomic molecule (most gases) $n = 2n_{mol}$

$$\tau[\text{hr}] = \frac{.474T[\text{K}]}{P[\text{nTorr}]\sigma[\text{barn}]}$$

Example, Fermilab Tevatron

$$p = 1000 \text{ GeV} \rightarrow \sigma_{coulomb} \text{ negligible}$$

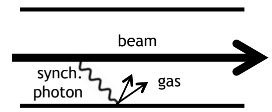
$$\sigma_{nuclear} = .4 \text{ barn, T} = 4 \text{ K}$$

$$20 \text{ hour lifetime} \rightarrow P < 2.3\times10^{-10} \text{ Torr}$$

Residual Gas Interaction (electrons)

$$\sigma = \sigma_{coulomb} + \sigma_{brem} \approx \sigma_{brem}$$

Can be complicated by "photodesorption" in which a synchrotron photon causes gas molecules to be desorbed from the beam pipe walls. This effectively increases the gas density



$$n_{eff} = n_0 + GN$$

Probability of desorption/electron

Initial density

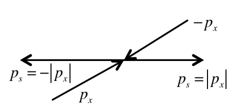
Intensity

$$P_{eff} = \frac{k_b T n_{eff}}{2}$$

Touschek Effect

Scattering of one particle by another in the bunch, which changes the energy of the particles enough to get lost.

If we look in the rest frame of the bunch



Maximum effect in lab frame

$$\Delta p_{max} = \gamma p_s = \gamma p_x = \gamma x' p$$

$$\frac{\Delta E_{max}}{E} = \beta^2 \frac{\Delta p_{max}}{p} = \beta^2 \gamma x'$$

If $\Delta E_{max} > \Delta E_a$ (energy acceptance), beam will be lost. Exact derivation is very complicated, but form is

$$\frac{1}{\tau_{Touschek}} = \frac{r_0^2 c N_b}{\gamma^4 \epsilon_x \epsilon_y \epsilon_L} \left\langle f\left(\beta_x, \beta_y, \delta, D_x, \frac{\Delta E_a}{E}\right)\right\rangle$$

Averaged over ring

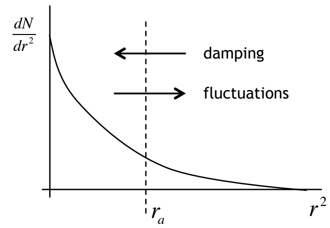
Quantum Lifetime (electron machines)

Quantum fluctuations due to photon radiation may cause a particle to exceed the energy acceptance of the machine (extreme case of emittance growth discussed earlier).

For a gaussian distribution of particles, we can write

$$-\frac{1}{2}dr^2 \qquad \frac{dN}{rdrd\phi} = \frac{N}{2\pi\epsilon}e^{-r^2/2\epsilon}$$

$$\oint d\phi = 2\pi \qquad \longrightarrow \frac{dN}{dr^2} = \frac{N}{2\epsilon}e^{-r^2/2\epsilon}$$



$$\tau_{r^2} = \frac{1}{2}\tau_x; \quad \tau_x = 2\frac{\tau_s E_s}{U_s}(1-\mathcal{D}) \quad \text{(see sync. lecture)}$$

$$\left.\frac{dN}{dt}\right|_{damping} = \left.\frac{dN}{dr^2}\right|_{r_a^2}\left.\frac{dr^2}{dt}\right|_{damping}$$

$$\left.\frac{dr^2}{dt}\right|_{damping} = -\left.\frac{r^2}{\tau_{r^2}}\right|_{r^2=r_a^2}$$

$$\Longrightarrow$$

$$-\left.\frac{r^2}{\tau_{r^2}}\right|_{r_w^2} = -2\frac{r_w^2}{\tau_x}$$

$$\left.\frac{dN}{dt}\right|_{r=r_a} = \left(-2\frac{r_a^2}{\tau_x}\right)\left(\frac{N}{2\epsilon}\right)e^{-r_a^2/2\epsilon}$$

$$= -N\frac{r_a^2}{\epsilon\tau_x}e^{-r_a^2/2\epsilon}$$

but at equilibrium
$$\frac{dN}{dt} = \left.\frac{dN}{dt}\right|_{fluctuations} + \left.\frac{dN}{dt}\right|_{dampng} = 0$$

$$\longrightarrow \left.\frac{dN}{dt}\right|_{fluctuations} = +N\frac{r_a^2}{\epsilon\tau_x}e^{-r_a^2/2\epsilon}$$

If, there's an aperture restriction, then $\left.\frac{dN}{dt}\right|_{damping} = 0 \rightarrow \frac{dN}{dt} = -\left.\frac{dN}{dt}\right|_{fluctuations}$

If this rate is small, then the shape will not change, so

$$\frac{dN}{dt} = -N\frac{r_a^2}{\epsilon_x\tau_x}e^{-r_a^2/2\epsilon_x} = -\frac{N}{\tau_q}$$

quantum lifetime

$$\tau_q = \tau_x\frac{\epsilon}{r_a^2}e^{+r_a^2/2\epsilon_x} = \tau_x\frac{\epsilon}{d^2}e^{+d^2/2\epsilon_x}$$

d is the limiting half aperture

Can do the same analysis in the longitudinal plane

$$\tau_q = \tau_\varepsilon\frac{\sigma_E^2}{\Delta E_a^2}e^{+\Delta E_a^2/2\sigma_E^2}$$

Luminosity

Once we know the loss mechanism, we have

$$\frac{dN_b}{dt} = -\mathcal{L}\sigma_{loss}$$

$$\mathcal{L} \propto N_b^2 \equiv kN_n^2$$

$$\frac{d\mathcal{L}}{dt} = 2\mathcal{L}N_b\frac{dN_b}{dt} = -2kN_b\mathcal{L}\sigma_{loss}$$

$$\equiv -\frac{\mathcal{L}}{\tau_{\mathcal{L}}}$$

Luminosity lifetime

$$\tau_{\mathcal{L}} = \frac{1}{2kN_b\sigma_{loss}} = \frac{N_b}{2\mathcal{L}\sigma_{loss}}$$

$$= \frac{\mathcal{L}}{N_b}$$

$$\text{initial } \tau_{\mathcal{L}} = \frac{N_0}{2\mathcal{L}_0\sigma_{loss}}$$

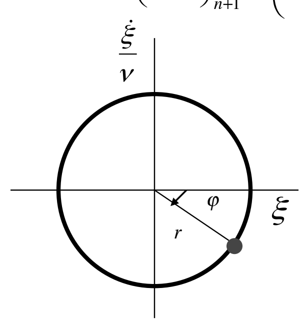
Emittance Growth

Elastic scattering $\quad x, x' \longrightarrow x, x' + \theta$

In Floquet coordinates $\quad \Delta\dot{\xi} = \nu\sqrt{\beta}\Delta x' = \nu\sqrt{\beta}\theta$

In one turn…

$$\begin{pmatrix} \xi \\ \dot{\xi} \end{pmatrix}_{n+1} = \begin{pmatrix} \cos 2\pi\nu & \frac{1}{\nu}\sin 2\pi\nu \\ -\nu\sin 2\pi\nu & \cos 2\pi\nu \end{pmatrix} \begin{pmatrix} \xi \\ \dot{\xi} + \nu\sqrt{\beta} \end{pmatrix}_n$$



$$\xi = r\cos\varphi$$

Using a different symbol

$$\dot{\xi} = -\nu\sin\varphi$$

$$r^2 = \xi^2 + \left(\frac{\dot{\xi}}{v}\right)^2$$

$$r^2\left(\dot{\xi} + \Delta\dot{\xi}\right) = \xi^2 + \left(\frac{\dot{\xi}}{v}\right)^2 + 2\frac{\dot{\xi}\Delta\dot{\xi}}{v^2} + \frac{\left(\Delta\dot{\xi}\right)^2}{v^2}$$

$$\longrightarrow \frac{\Delta r^2}{dn} = \frac{1}{v^2}\left(2\dot{\xi}\Delta\dot{\xi} + \left(\Delta\dot{\xi}\right)^2\right)$$

$$= -2\sqrt{\beta}r\sin\varphi\theta + \beta\theta^2$$

averages to zero

$$\frac{\Delta\left\langle r^2\right\rangle}{dn} = \beta\left\langle\theta^2\right\rangle$$

The mean angle from Coulomb scattering is

$$\left\langle\theta^2\right\rangle = \left(\frac{13.6\text{ MeV}}{\beta pc}\right)^2 \frac{s}{X_0}$$

radiation length

So for residual gas

$$\frac{d\left\langle r^2\right\rangle}{dn} = \left\langle\beta\right\rangle\left(\frac{13.6\text{ MeV}}{\beta pc}\right)^2 \frac{C}{X_0}$$

circumference

For electron machines, $\tau_{scatt} >> \tau_{damping}$, so this is only a factor for hadron machines

Other sources of scattering

1. dipole power supplies

$$\left\langle\theta^2\right\rangle = \left\langle\frac{(\Delta B)^2 L^2}{(B\rho)^2}\right\rangle$$

2. Ground motion or vibration (random quad motion)

$$\left\langle\theta^2\right\rangle = \left\langle\frac{(\Delta x)^2}{f^2}\right\rangle$$

Recall

$$\epsilon_{RMS} = \frac{\langle r^2 \rangle}{2}$$

$$\frac{d\epsilon}{dt} = \frac{1}{\tau_s}\frac{d\langle r^2 \rangle}{dn} = \frac{1}{2\tau_s}\langle \beta\theta^2 \rangle$$

Intrabeam scattering is also very important, but too complicated to discuss here