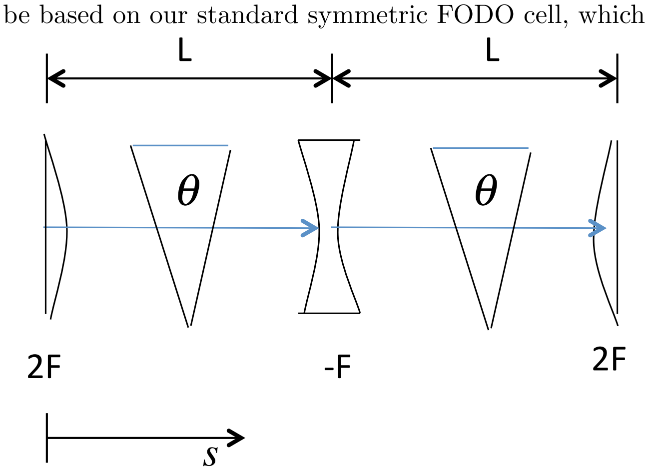
USPAS Accelerator Physics Final Exam

January 30, 2014

General Guidelines

- This is an "open book" exam. You may use the text, lectures, homeworks, or any of the recommended resources, including last year's final. You are expected to work independently and to not seek out other sources for the solutions.
- There are a total of four problems, which do not have equal weight.
- You may use anything that appeared in the lectures, textbook or assigned homework, without re-deriving it.
- Full or partial credit will only be given if your reasoning can be followed, so show your work. Please give answers in the requested [units] when specified.
- The exam is due at 9AM tomorrow. Late exams will have their score reduced by 10%, with an additional 10% deducted for each additional hour.
- All problems are straightforward applications of what you have learned. There are no trick questions or complex calculations. If you find yourself working hard, it's a good sign you're not doing the problem correctly.
- If you think there's a problem with the test, contact Eric at 630-336-1893 or prebys@fnal.gov. Any necessary corrrections or clarifications will be sent to the email list and posted on the web page, so check both frequently!

The first three problems will be based on our standard symmetric FODO cell, which we have seen many times:



where each cell contains focusing and defocusing quadrupoles of focal lengths (in the horizonal plane) of F and $-F$, respectively, spaced L apart. The entire length between the quadrupoles is taken up by identical bend dipoles, each of which bends the beam by an angle θ. Positions s within the cell are measured from the center of the first focusing quadrupole. You may use the thin lens approximation for the quadrupoles, assume each (half period) dipole is $\approx L$ long, ignore any fringe field effects of the dipoles, and assume there are no horizontal dipole fields.

Problem 1

a. Write expressions for all Twiss parameters in the bend plane (α_x,β_x, and γ_x) at middle of the the first bend ($s = L/2$). Express your answers in terms of (as appropriate) β_{max}, β_{min}, F, and L.

b. If I want the phase advance across one cell to be *exactly* $60°$, what value of F should I use, in terms of L? Simplify your answer as much as possible.

c. I build a ring out of N_{cell} of these identical FODO cells. What is the horizontal tune ν_x of the machine?

d. Write an expression for the bend angle θ per (half cell) dipole in terms of N_{cell}.

e. Assuming that my ring contains small anomalous magnetic errors up to and including sextupole terms, what are the values of the fractional tune I should avoid?

f. In light of this, and my $60°$ phase advance, write an expression or expressions for the *allowed* values of N_{cell}.

Problem 2

Now let's put in some numbers. I design a ring to accelerate protons from kinetic energy of $K_{inj} = 2\text{GeV}$ to $K_{ext} = 20\text{GeV}$. I make each FODO cell a *total* of 30m long. Assume I still want a phase advance of $\mu = 60°$ per cell.

a. What are the values for (relativistic) β, momentum p[GeV/c], and beam rigidity $(B\rho)$[T-m] at injection and extraction?

b. What value of focal length F[m] do I need?

c. If I use 2m long quadrupoles for the focusing magnets, what is the required gradient B'[T/m] when the beam is at the highest energy?

d. I choose the most stable value of N_{cell} between 26 and 30. Based on your answer to 1(f), what is it?

e. What is the numerical value for the horizontal tune ν_x?

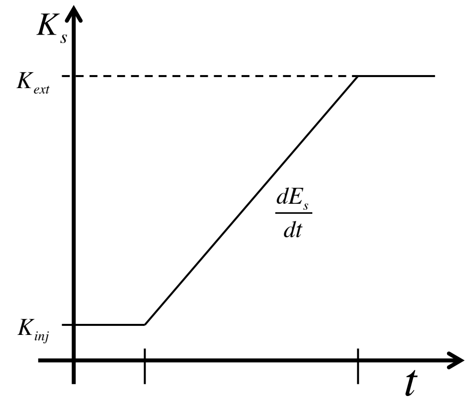
f. Give numerical values for maximum values of the betatron function β[m] and and the dispersion D[m].

g. If my (95%) normalized beam emittance is $\epsilon_{95} = 10\pi$-mm-mr, what is the maximum RMS beam size in x, (σ_x) [mm] during injection, assuming for the moment that there is negligible momentum spread?

h. Let's assume that at at injection, I want the RMS contribution to the beam size due to the momentum spread at the point of maximum dispersion to be roughly the same as the contribution to the beam size due to the betatron oscillations in the previous part. What's the maximum allowable RMS momentum spread $\Delta p/p_0 = \delta$ I can have? What does this correspond to in terms of energy distribution σ_E[MeV]?

Problem 3

Moving on to the longitudinal plane... Start with the synchrotron in the previous problem. Assume that we are going to inject at a kinetic energy $K_{inj} = 2$GeV, accelerate at a linear rate to $K_{ext} = 20$GeV, stop accelerating and extract it; that is, assume we are not accelerating at the time beam is injected and extracted.



a. If I design an RF system with a harmonic of $h = 120$, what are the injection and extraction frequencies f_{inj} and f_{ext}[MHz].

b. What are the injection and extraction slip factors η_{inj} and η_{ext}? (you may assume that $\gamma_t \approx \nu_x$)

c. I want to have at least a factor of 4 overhead in the bucket height ΔE_b over the RMS energy distribution of the beam at injection; that is $\Delta E_b = 4\sigma_E$. What value of the longitudinal beta function β_L [s/eV] does my RF system need to have if my energy spread just satisfies the condition specified at the end of problem 2?

d. If this is my value of β_L, calculate the following *at injection*:

- The synchrotron tune ν_s
- The total required RF voltage V_0 [MV]

e. I advance the accelerating phase to begin accelerating the beam. If I can set my synchronous phase ϕ_s in $5°$ increments ($5°$, $10°$, $15°$, etc), what is the maximum value for ϕ_s[°] which will keep my bucket overhead *at least* a factor of 3 during initial acceleration? This problem must be solved numerically. (Hint: you've already solved the equation for the case of no acceleration. Isolate the dependence on ϕ_s and plug in increasing values.)

f. Given these values of V_0 and ϕ_s, what is initial acceleration ramp dE_s/dt[GeV/s]?

Problem 4

Consider a proton-proton collider with the following parameters:

- Beam energy: E_b $(\gg m_p c^2)$
- Circumference: C
- Number of bunches: n_b
- Number of protons ber bunch: N_b
- Normalized RMS emittance: $\epsilon_x = \epsilon_y = \epsilon_N$.
- A single, low beta interaction region.
- Final focusing triplets are located a distance d away from the interaction point.

Assume that we are running at the maximum luminosity L_0, limited by:

- A fixed normalized beam emittance ϵ_N.
- The beam-beam tune shift ξ.
- The aperture A of the final focusing quads ($A \equiv$ diameter of the beam pipe)
- The minimum bunch spacing; that is, n_b is a maximum.

In terms of the above parameters, answer the following questions:

a. Write an expression for the betatron function β_A in the final focusing triplet, assuming I want at least a 6σ clearance for the beam (assume $\beta_x \approx \beta_y$ at this point)?

b. What is the approximate minumum value for the beta function β^* at the collision point? You may assume $\beta^* \ll d$. (Hint: How does the beta function behave as I move away from the minimum at the collision point?)

c. If I double the energy, but keep the circumference C the same (that is, I double the strength of my magnets), by what factor will the maximum luminosity increase, assuming I can change β^*, but that I am subject to all the same limitations itemized above.